

Numerical Study of Quantum Noise of AlGaAs Fabry Perot Semiconductor Lasers Operating in Single-Mode

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ABSTRACT

Semiconductor lasers are used as light sources in CD/DVD players, optical communication systems and other optical devices. But they often involve various noise and instability problems due to fluctuations of photon and electron numbers. In this paper, a numerical study has been done to understand the quantum noise characteristics of a semiconductor laser. Quantum noise is an intrinsic property of semiconductor lasers and impossible to control in principle. Hence analysis of noise characteristics for solitary lasers is important for noise largely limits the performance of the device. The rate equations for photon number and carrier number have been solved by considering self suppression coefficient of the laser. We consider Langevin noise sources for both photon number and carrier number to demonstrate the photon and carrier fluctuation. The rate equations are applied to typical 780-nm AlGaAs Fabry Perot semiconductor lasers operating in single mode by optimally choosing all the required parameters. Matlab is used to perform the numerical simulation. We analyze time-varying profiles of the fluctuating photon and carrier numbers for different injection current. The quantum noise of laser is calculated and presented through relative intensity noise (RIN) of the laser output. Frequency spectrum of the intensity noise is calculated with the help of fast Fourier transform (FFT). Transient behavior of the laser output is also demonstrated to understand noise characteristics of the device. Correspondence between the simulation data and practical result is also found.

Keywords: Quantum noise, relative intensity noise, semiconductor laser, single-mode, Langevin noise, photon fluctuation, rate equation.

1. Introduction

Semiconductor lasers are widely used as coherent light sources for a variety of applications including fiber optics communication systems and CD/DVD drive [1]. Quantum mechanical effects often limit the application of semiconductor lasers. Early calculations of quantum noise were based on small-signal analysis. This concept was developed by McCumber [2] and was applied to semiconductor lasers by Haug [3]. Information concerning the instantaneous fluctuations of the photon and carrier numbers was missed in such small-signal calculations. Direct numerical integration of the rate equations has been applied to overcome the limitations of the small-signal analysis [4]-[14].

In this paper, we have developed self consistent rate equations of semiconductor lasers operating in single-mode for numerical simulation and analysis of quantum noise. The model has been applied to a solitary 780-nm AlGaAs laser assuming that only the fundamental transverse mode exists. The rate equations of the modal photon number $S(t)$ and number of injected electrons $N(t)$ are used to study the modal dynamics and quantum noise characteristics of semiconductor lasers. Langevin noise sources for photon number and carrier number are included in the rate equations to consider fluctuated spontaneous emission. Frequency spectra of the intensity noise are calculated with the help of the fast Fourier transform (FFT).

The theoretical model of our analysis is developed in the next section. It talks about the rate equations for photon

number and carrier number and also about the introduction of noise sources for photon number and carrier number. In section III we describe the algorithm for numerical simulation. In the very next section, we discuss the various results of numerical simulation for 780-nm AlGaAs laser and analyze them. We also present few practical results that we get in the laboratory in this section. Lastly, we conclude this paper with some concluding remarks on the results in the last section.

2. The Rate Equation Model

The rate equations for semiconductor lasers operating in single-mode can be derived as follows [15].

For photon number:

$$\frac{dS}{dt} = (G - G_{tho})S + \frac{a\xi N}{V} + F_S(t) \quad (1)$$

For injected carrier (electron) number:

$$\frac{dN}{dt} = -AS - \frac{N}{\tau_s} + \frac{I}{e} + F_N(t) \quad (2)$$

where $G=A \cdot BS$ is the gain of single mode laser with wavelength λ ,

G_{tho} is the threshold gain of solitary laser given by

$$G_{tho} = \frac{c}{n_r} \left[k + \frac{1}{2L} \ln \frac{1}{R_f R_b} \right] \quad (3)$$

The noise sources in this model are $F_S(t)$ and $F_N(t)$, which are considered to be the Langevin noise sources for photon number and carrier number.

$$F_S(t) = \sqrt{\frac{V_{SS}(t)}{\Delta t}} g_S(t) \quad (4)$$

$$F_N(t) = \sqrt{\frac{V_{NN}(t) + KV_{SN}(t)}{\Delta t}} g_N(t) - KF_S(t) \quad (5)$$

The Langevin noise sources are considered to be functions in inducing instantaneous fluctuations in photon number and carrier number due to spontaneous emission and recombination process. $F_S(t)$ and $F_N(t)$ can be approximated as Gaussian distribution with zero mean value.

The variances $V_{SS}(t)$, $V_{NN}(t)$ and $V_{SN}(t)$ are given by

$$V_{SS}(t) = \left\{ \frac{\xi a}{V} (N + N_g) + G_{tho} \right\} S + \frac{\xi a N}{V} \quad (6)$$

$$V_{NN}(t) = \left\{ \frac{\xi a}{V} (N + N_g) S \right\} + \frac{N}{\tau S} + \frac{I}{e} \quad (7)$$

$$V_{SN}(t) = -\frac{\xi a}{V} \left\{ (N + N_g) S + N \right\} \quad (8)$$

The other parameters are

A is the linear gain given by

$$A = \frac{\xi a}{V} (N - N_g) \quad (9)$$

B is the self-suppression coefficient written as

$$B = \frac{9}{4} \frac{\hbar \omega}{\epsilon_0 n_r^2} \left(\frac{\xi \tau_{in}}{\hbar V} \right)^2 a R_{cv}^2 (N - N_s) \quad (10)$$

$$K = -\frac{V_{SN}(t)}{V_{SS}(t)} \quad (11)$$

In the above equations, a is the differential gain coefficient, ξ is the field confinement factor, V is the volume of the active region, τ_s is the injected carrier (electron) lifetime, I is the injection current, e is the electron charge, k is the internal loss in the laser cavity, N_g is the electron number at transparency, b is the width of the linear gain coefficient, \hbar is the reduced Planck constant, τ_{in} is the intra-band relaxation time, R_{cv} is the dipole moment and N_s is the electron number characterizing self-suppression coefficient.

In (4) and (5), g_s and g_N are Gaussian random variables in the range of

$$-1 \leq g_s \leq 1 \text{ and } -1 \leq g_N \leq 1 \quad (12)$$

The Gaussian random variables are generated using the Box-Muller transformation [15] method in which two uniformly distributed random numbers u_1 and u_2 are taken in the range between -1 to +1. The following equations are then used to obtain g_s and g_N .

$$g_s(t) = \frac{1}{5} \left\{ \sqrt{-2 \log u_1(t)} \cos[2\pi u_2(t)] \right\} \quad (13)$$

$$g_N(t) = \frac{1}{5} \left\{ \sqrt{-2 \log u_1(t)} \sin[2\pi u_2(t)] \right\} \quad (14)$$

The photon number fluctuation is defined as

$$\delta S(t) = S(t) - \bar{S} \quad (15)$$

where \bar{S} is the time average photon number.

In this analysis, the time fluctuating components are transformed into Fourier frequency components. The noise is then calculated from the Fourier frequency components. Relative intensity noise (RIN) is calculated from the photon number fluctuation in (15) that can be obtained from the time integration of the rate equations for photon number.

$$RIN = \frac{1}{\bar{S}^2} \frac{\Delta t^2}{T} |FFT(\delta S(t))|^2 \quad (16)$$

3. Numerical Simulation

Our aim here is to obtain the instantaneous photon number $S(t)$, carrier number $N(t)$ and corresponding relative intensity noise through numerical simulation. The typical values for 780-nm AlGaAs laser parameters considered for numerical simulation are listed in Table 1.

The fourth-order Runge-Kutta algorithm has been used to solve the rate equations to obtain the results. For the numerical integrations, a short time interval of $\Delta t = 5\text{ps}$ has been used. Such a small value of Δt produces noise sources that can approximately describe a white noise spectrum up to a frequency of 200GHz which is enough to get the signal at relaxation oscillation frequency. Extending the integration to a time period as long as $5\mu\text{s}$ allows us to demonstrate the intensity noise as low as 200kHz.

Table 1: Typical values of 780-nm AlGaAs laser parameters [15]

Symbol	Definition	Value	Unit
a	tangential gain coefficient	2.75×10^{-12}	$\text{m}^3 \text{s}^{-1}$
$ R_{cv} ^2$	Squared value of absolute value of dipole moment	2.8×10^{-57}	$\text{C}^2 \text{m}^2$
ξ	confinement factor of field	0.2	-
τ_{in}	electron intraband relaxation time	0.1	ns
τ_s	average electron lifetime	2.79	ns
N_s	electron number characterizing non-linear gain	1.7×10^8	-
N_g	electron number at transparency	1.89×10^8	-
V	volume of the laser active region	75	μm^3
d	thickness of the laser active region	0.11	μm
L	length of the laser active region	300	μm
n_r	refractive index of laser active region	3.59	-
k	internal loss in the laser cavity	10	cm^{-1}
R_f	reflectivity of front facet	0.2	-
R_b	reflectivity of back facet	0.7	-

4. Results and Discussion

The proposed model is applied to 780-nm AlGaAs lasers to study the quantum noise while the laser is operating in single-mode. The characteristics of noise are expressed in

terms of relative intensity noise (RIN) and observed for different injection currents for the operating laser. In Fig. 1, we observe different values of RIN (quantum noise) for different values of injected current to the laser. Here, the solid lined curve indicates RIN characteristics for $I=1.7I_{th}$ where I_{th} is the threshold current of the laser after which the lasing starts and dotted lined curve indicates RIN characteristics for $I=1.2I_{th}$. From the figure, it is seen that for a lower value of injection current which is 1.2 times of the threshold current, the RIN is greater; while for the current being 1.7 times of the threshold, RIN is comparatively low. Also the relaxation oscillation frequency is shifted to the right that is, to the higher frequency. The RIN values are around 10^{-16}Hz^{-1} in the lower frequency region up to 100MHz. The relaxation oscillation frequency is 2GHz for the lower current and 3GHz for the higher current. In both cases, RIN is observed because of the photon number fluctuation.

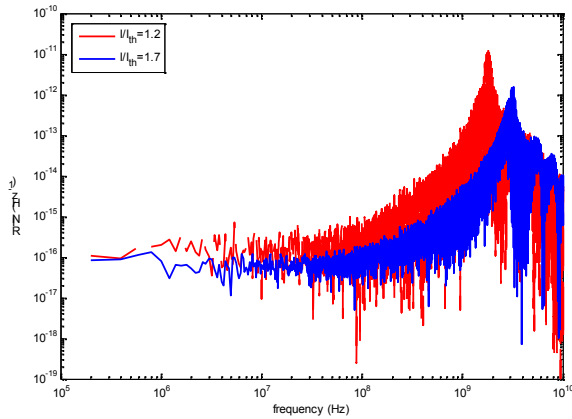


Fig. 1: Frequency spectrum of quantum RIN at $I/I_{th}=1.2$ (dotted line) and $I/I_{th}=1.7$ (solid line). Relaxation oscillation peak shifts to the right and RIN value decreased with injection current.

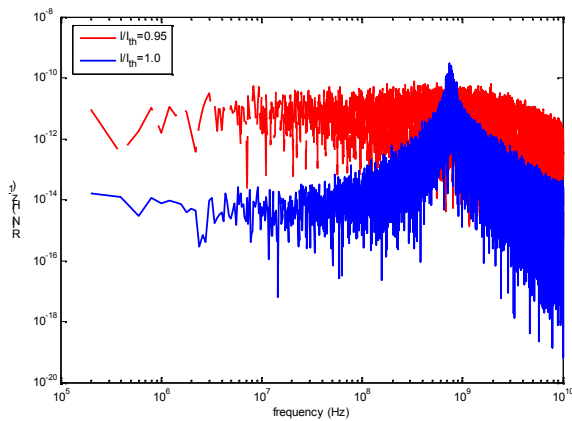


Fig. 2: Quantum RIN spectrum (before and after lasing operation) for different injection current with $I/I_{th}=0.95$ (dotted line) and $I/I_{th}=1.0$ (solid line). Before lasing operation the RIN is due to carrier number fluctuation and after lasing started the source of RIN is photon number fluctuation.

In Fig. 2, we observe the RIN characteristics for two different injection current. Here the solid lined curve indicates RIN characteristics for $I=1.0I_{th}$ when the lasing

operation has just been started and the dotted lined curve indicates RIN characteristics for $I=0.95I_{th}$ before the start of lasing operation. From the figure, when the injection current is equal to the threshold current, the RIN is lower compared to the RIN that is produced by injection current less than the threshold current. For current less than the threshold no oscillation occurs, hence the RIN characteristic shows no oscillation frequency peak. But, for the current at and above threshold, figure shows oscillation frequency peak. Therefore, the relaxation oscillation frequency is found for $I=I_{th}$ at around 800MHz. Although for $I=0.95I_{th}$ there is no oscillation, RIN presents is greater than that for $I=1.0I_{th}$. This noise comes from the carrier (electron) number fluctuation, not from photon number fluctuation. In case of $I>I_{th}$, at any particular frequency, lower current exhibits greater noise than the higher current all the way through the operation.

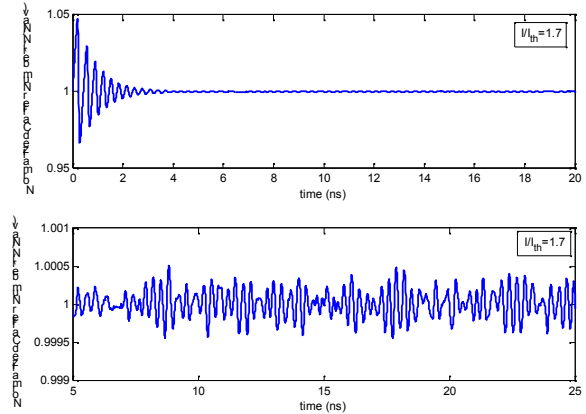


Fig. 3: Transient response of electron/carrier number which is normalized with average carrier number $N_{av}=2.68\times 10^8$. After 4ns the transient dies out and carrier number fluctuates around its average value.

Fig. 3 shows the time variation of carrier response in the lasing phenomena. Initially carrier number variation shows transient response at $0\text{ns}<\text{time}<4\text{ns}$ before laser output becomes stable. After that when laser output becomes stable transient response to the carrier numbers dies out, but carrier number still fluctuates around its average value due to Langevin noise.

Fig. 4 shows the time variation of photon response in the lasing phenomena. Initially photon number variation shows transient response at $0\text{ns}<\text{time}<4\text{ns}$ before laser output becomes stable. At stable region, laser output still shows fluctuation around its average value due to Langevin noise. Dependence of photon population on carrier population can be explained as follows. During switch on process of the laser, the injected carrier concentration rises significantly above its equilibrium level. This causes photon population to exceed its equilibrium level, which in turn results transient oscillation.

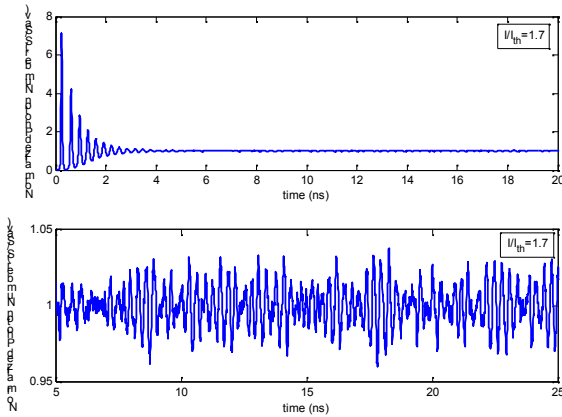


Fig. 4: Time varying profile of photon number which is normalized with average photon number $S_{av}=2.28 \times 10^5$. After transient dies out the photon number still fluctuates around its average value that is the source of the quantum noise.

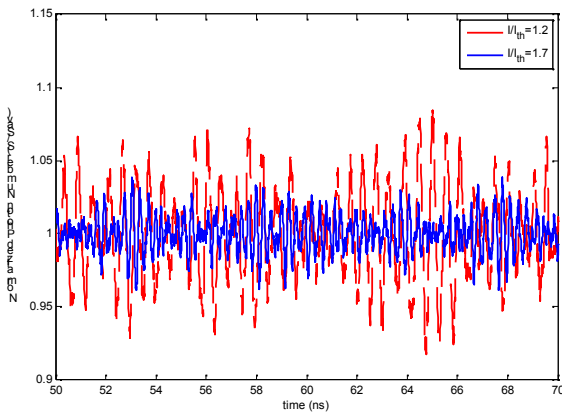


Fig. 5: Time varying profile of photon number after transient dies out for two different injection current with $I/I_{th}=1.2$ (dotted line) and $I/I_{th}=1.7$ (solid line). With high injection current photon number fluctuation is decreased with increased frequency.

The dotted lined curve in Fig. 5 shows photon number fluctuation with time for $I=1.2I_{th}$ and solid lined curve is that for $I=1.7I_{th}$. The photon fluctuation is higher for $I=1.2I_{th}$ and is lower for $I=1.7I_{th}$. Hence, lower current exhibits higher instability than high current injection. This

suppression of fluctuations leads to a decrease in the level of RIN with increasing I .

The plotted photon number fluctuations are far from the relaxation regime. The plot features that the repetition of fluctuation becomes faster with increasing injection current I , which indicates that the relaxation oscillation frequency has increased. That's why the relaxation oscillation peak has been shifted to the higher frequency for higher injection current as shown in Fig. 1.

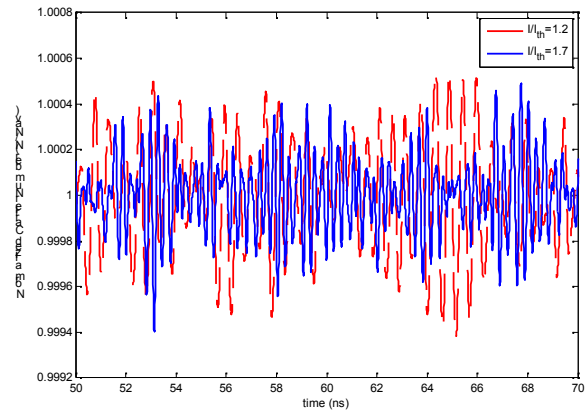


Fig. 6: Carrier number fluctuation with time after transient dies out with $I/I_{th}=1.2$ (dotted line) and $I/I_{th}=1.7$ (solid line). Though the fluctuation remains same with injection current the frequency is increased like photon number variation.

Fig. 6 shows comparative electron number fluctuation for two different injection current. The solid lined curve is the carrier fluctuation for $I=1.2I_{th}$ and dotted lined curve is that for $I=1.7I_{th}$. The electron fluctuation is higher for the lower injection current of $I=1.2I_{th}$ and that for $I=1.7I_{th}$ remains almost same though photon number fluctuation is decreased with the increase of injection current value. Also, carrier number fluctuation is 100 times less than the photon number fluctuation. Therefore, we can conclude from Fig. 5 and 6 that the quantum noise generated after lasing has started is due to photon number fluctuation, not for carrier number fluctuation.

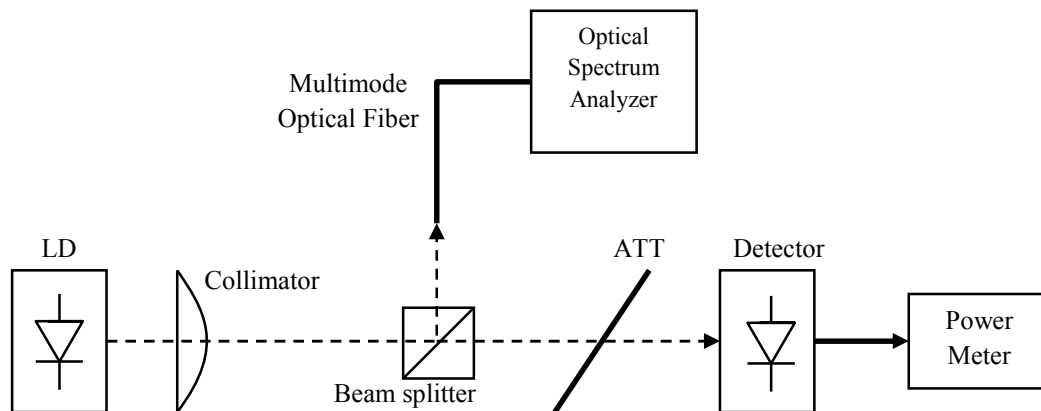


Fig. 7: Practical setup to measure optical power and intensity noise of laser diodes.

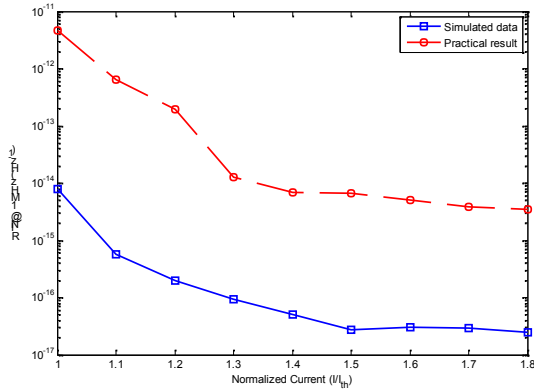


Fig. 8: Comparison of simulated data (solid line) with practical result (dotted line) for the variation of quantum RIN at 1MHz with normalized current injection. Noise level for simulated data is 20dB less than the practical data as we do not consider added noise of the electric circuit and other components.

The variation of RIN value at a particular noise frequency of 1MHz with injection current is shown in Fig. 8. The dotted line shows the simulated data and solid line represents practical results. The quantum noise reveals maximum value at the threshold current and reduces with increasing of the injection current. We observe that both simulated and practical data has excellent correspondence though the noise level for simulated data is 20dB less than the practical data. The difference may be due to the different selection of parameters for the laser material and structure as well as additional fluctuation phenomena present in the real device. [16]

5. Conclusion

In this paper, the quantum noise has been studied numerically for solitary AlGaAs semiconductor laser operating in single-mode. The rate equations for photon number and carrier number of solitary semiconductor lasers are applied to typical 780-nm AlGaAs lasers through optimally choosing the necessary parameters. Self-suppression coefficient of photon number is considered in the rate equations. Also, Langevin noise sources for both photon number and carrier number have been considered to analyze the fluctuations present in lasing output. Different current has been applied for the lasing operation to demonstrate the effect of injection current on laser.

The objective of this paper is to study the characteristics of intensity noise present in the laser output that degrades the laser performance and hence limits its operation as an optical device. For this we numerically simulate the rate equations for AlGaAs lasers using Matlab. The lasing output noise is calculated as RIN (relative intensity noise). Results show that lower injection currents produce higher quantum noise due to higher fluctuations of photon numbers present, though the carrier number fluctuation remains same for different current injection values. We found that the RIN value is decreased with increased current injection. Before threshold the laser does not show any oscillation, but the output still shows higher RIN values. We observe that this

noise comes from the carrier fluctuation, not from photon fluctuation. We also found that initially lasers show transient response that dies out after few ns before lasers become stable. At stable operation lasers still show higher quantum noise at lower frequency due to fluctuation in the photon numbers.

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