Journal of Statistical Research *ISSN 0256 - 422 X*
2005, Vol. 39, No. 2, pp. 127–134
Bangladesh

ESTIMATION OF VARIANCE IN AN ANOVA SETUP

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SUMMARY

In an ANOVA setup one tests the global null hypothesis against the alternative that at least one pair of means differ. In this paper we consider the estimation of the variance, when it is suspected, but one is not sure, that the null hypothesis holds. We consider the (i) unrestricted unbiased estimator (UUE), (ii) unrestricted biased estimator (UBE), (iii) restricted unbiased estimator (RUE), (iv) restricted biased estimator (RBE), (v) preliminary test estimator (PTE) using UUE and RUE, (vi) Stein-type estimator (SE) using UBE and RBE of variance. We derive the bias and risk expressions for these estimators to compare them. It is shown that Stein-type estimator (SE) dominates uniformly over the UUE as well as the PTE when the critical value for preliminary test is 1.

Keywords and phrases: Point estimation, preliminary test estimator, Stein-type estimation, quadratic risk

1 Introduction

Stein's (1956–64) prolific and innovative ideas enriched mathematical statistics in the direction of point and set estimation of parameters among many other topics. For the multiparameter problem Stein (1956) proved that the usual MLE or LSE is inadmissible under a quadratic loss function and James-Stein (1961) became the symbol of the paradox which is a non-linear estimator depending on a test-statistic to test some plausible null hypothesis. For example, in the case of several mean problem, the test-statistics relates to the test of equality of the means.

For the estimation of the variance of a normal distribution, Stein's (1964) theory really boils down to a preliminary test estimator (PTE) when it is suspected that the mean of the distribution is zero. The preliminary test estimator was first proposed by Bancroft (1944). It stands out as a precursor to the Stein-estimator, but soon became important due to the Stein-estimator of variance. A detailed survey and the importance of Stein's method of variance estimation is given in the classic paper "Developments in Decision-Theoretic Variance Estimation" by Maatta and Casella (1990). The paper details out various aspects of variance estimation based on a single sample. Before the publication of this paper many other papers dominated the area such as Brown (1968), Rukhin (1987), and Strawderman (1974) among others. In this paper, we consider the problem of estimation of variance based on multiple samples of varying sizes from different normal distributions with the same variance but different means, which are suspected, but not sure to be equal.

2 Various Estimates of σ^2

Consider the ith sample $\{x_{i1},\dots,x_{in_i}\}$ of size n_i from the normal distribution $\{N(\theta_i,\sigma^2)|i=1,2,\dots,p\}$. Let $\bar{x}_i$ be the ith sample mean and S_i^2 be the ith sample *unbiased* estimator of σ^2 i.e. $(n_i-1)S_i^2=(x_{i1}-\bar{x}_i)^2+\cdots+(x_{in_i}-\bar{x}_i)^2$ $i=1,\dots,p$. An *unrestricted unbiased estimator* (UUE) of σ^2 is then defined by

$$S_U^2=m^{-1}\{(n_1-1)S_1^2+\cdots+(n_p-1)S_p^2\},\quad m=n-p \tag{2.1}$$

where $n=n_1+\cdots+n_p$. Clearly, $E[S_U^2]=\sigma^2$. If now, $\theta_1=\ldots=\theta_p=\theta_0$ (unknown) holds, then the *restricted unbiased estimator* (RUE) is defined by

$$S_R^2=(m+q)^{-1}\left\{\sum_{j=1}^{n1}(x_{1j}-\bar{\bar{x}})^2+\cdots+\sum_{j=1}^{n_p}(x_{pi}-\bar{\bar{x}})^2\right\}, \tag{2.2}$$

where $q=p-1$ and $\bar{\bar{x}}=n^{-1}(n_1\bar{x}_1+\cdots+n_p\bar{x}_p)$. Note that

$$(m+q)S_R^2=mS_U^2+(\bar{\mathbf{x}}-\bar{\bar{x}}\mathbf{1}_p)'\mathbf{N}(\bar{\mathbf{x}}-\bar{\bar{x}}\mathbf{1}_p) \tag{2.3}$$

where

$$\bar{\mathbf{x}}=(\bar{x}_1,\dots,\bar{x}_p)',\quad \mathbf{1}_p=(1,\dots,1)'-\text{a } p\text{-tuple of 1's}\quad\text{and}\quad \mathbf{N}=\text{Diag}(n_1,\dots,n_p). \tag{2.4}$$

Further, consider the likelihood ratio test-statistic for the null hypothesis $H_0:\theta_1=\cdots=\theta_p=\theta_0$ (unknown) given by

$$\mathcal{L}_n=(\bar{\mathbf{x}}-\bar{\bar{x}}\mathbf{1}_p)'\mathbf{N}(\bar{\mathbf{x}}-\bar{\bar{x}}\mathbf{1}_p)/q\,S_U^2 \tag{2.5}$$

which follows a central F–distribution with (q,m) d.f. under H_0 and non-central F–distribution with (q,m) d.f. and noncentrality parameter $\Delta^2=(\boldsymbol{\theta}-\theta_0\mathbf{1}_p)'\mathbf{N}(\boldsymbol{\theta}-\theta_0\mathbf{1}_p)\sigma^{-2}$. Under H_0, $E[S_R^2]=\sigma^2$.

Now, consider the following two additional estimators of σ^2 which are *biased unrestricted* (BUE) and *restricted biased* (RBE) estimators respectively

$$\tilde{\sigma}_U^2=\frac{mS_U^2}{m+2}\quad\text{and}\quad\hat{\sigma}_R^2=\frac{(m+q)S_R^2}{m+q+2}. \tag{2.6}$$

Let $F_{q,m}(\alpha)$ be the α–level critical value of the central F–distribution with (q,m) d.f. Then, a preliminary test estimator of σ^2 is defined by

$$S^2_{\text{PT}[1]}=\psi_1(\mathcal{L}_n)mS_U^2 \tag{2.7}$$

where

$$\psi_1(\mathcal{L}_n) = \frac{1}{m} I\big(\mathcal{L}_n \geq F_{q,m}(\alpha)\big) + \frac{1 + \frac{q}{m}\mathcal{L}_n}{m+q} I(\mathcal{L}_n < F_{q,m}(\alpha)). \tag{2.8}$$

Hence,

$$S^2_{\mathrm{PT}[1]} = S^2_U\, I(\mathcal{L}_n \geq F_{q,m}(\alpha)) + S^2_R I(\mathcal{L}_n < F_{q,m}(\alpha)) \tag{2.9}$$

Similarly, we define

$$S^2_{\mathrm{PT}[2]} = \psi_2(\mathcal{L}_n) m S^2_U \tag{2.10}$$

where

$$\psi_2(\mathcal{L}_n) = \frac{1}{m+2} I(\mathcal{L}_n \geq F_{q,m}(\alpha)) + \frac{1 + \frac{q}{m}\mathcal{L}_n}{m+q+2} I(\mathcal{L}_n < F_{q,m}(\alpha)). \tag{2.11}$$

Notice that $\psi_2(\mathcal{L}_n) \leq \psi_1(\mathcal{L}_n)$ and $S^2_{\mathrm{PT}[1]}$, is based on S^2_U and S^2_R and $S^2_{\mathrm{PT}[2]}$ is based on $\tilde{\sigma}^2_U$ and $\hat{\sigma}^2_R$ respectively. Further, if we choose $F_{q,m}(\alpha^*) = \frac{m}{m+2}$ i.e. $\alpha^* = F^{-1}_{q,m}\left(\frac{m}{m+2}\right)$, we obtain the special PTE called the Stein-type estimator, namely,

$$S^2_{[s]} = \phi_S(\mathcal{L}_n) m S^2_U \tag{2.12}$$

where

$$\phi_S(\mathcal{L}_n) = \frac{1}{m+2} I\left(\mathcal{L}_n \geq \frac{m}{m+2}\right) + \frac{1 + \frac{q}{m}\mathcal{L}_n}{m+q+2} I\left(\mathcal{L}_n < \frac{m}{m+2}\right). \tag{2.13}$$

3 Bias and Risk Expressions

In this section we consider the bias and the risk expressions of the various estimators of σ^2. First we consider the bias expressions:

(1) $b(S^2_U) = E(S^2_U - \sigma^2) = 0.$

(2) $B_2(S^2_R) = E[S^2_R - \sigma^2] = -\frac{\sigma^2}{m+q}(q - \Delta^2).$

(3) $b(\tilde{\sigma}^2_U) = E\big(\tilde{\sigma}^2_U - \sigma^2\big) = -\frac{2\sigma^2}{m+2}.$

(4) $b\big(\hat{\sigma}^2_R\big) = E(\hat{\sigma}^2_R - \sigma^2) = \frac{\sigma^2}{m+q+2}\big((m+2) + \Delta^2\big).$

(5) $b(S^2_{\mathrm{PT}[1]}) = -\frac{q\sigma^2}{m+q}\Big\{G_{q,m+2}\left(\left(1 + \frac{2}{m}\right)F_{q,m}(\alpha); \Delta^2\right) - G_{q+2,m}\big(\frac{q}{q+2}F_{q,m}(\alpha); \Delta^2\big)\Big\}$

$+\sigma^2\Delta^2 G_{q+4,m}\left(\frac{q}{q+4}F_{q,m}(\alpha; \Delta^2\right).$

(6) $b\big(S^2_{\mathrm{PT}[2]}\big) = -\frac{2\sigma^2}{m+2} - \frac{qm\sigma^2}{(m+2)(m+q+2)} G_{q,m+2}\left(\big(1+\frac{2}{m}\big)F_{q,m}(\alpha);\Delta^2\right) + \frac{\sigma^2}{m(m+q+2)}\times$
$\Big\{qG_{q+2,m}\big(\frac{q}{q+2}F_{q,m}(\alpha);\Delta^2\big) + \Delta^2 G_{q+4,m}\Big(\frac{q}{q+4}F_{q,m}(\alpha);\Delta^2\Big)\Big\}.$

(7) $b\big(S^2_{[s]}\big) = -\frac{2\sigma^2}{m+2} - \frac{qm\sigma^2}{(m+2)(m+q+2)} G(1;\Delta^2) + \frac{\sigma^2}{m(m+q+2)}\Big\{qG_{q+2,m}\big(\frac{q}{q+2}\frac{m}{m+2};\Delta^2\big)$
$+\Delta^2 G_{q+4,m}\big(\frac{q}{q+4}\frac{m}{m+2};\Delta^2\big)\Big\}.$

Now consider the risk expressions of the estimators under the loss function

$$L(\sigma^{*2};\sigma^2) = \frac{1}{\sigma^4}(\sigma^{*2}-\sigma^2)^2.$$

The risk-expressions corresponding to the estimators are given by

(1) $R_1(S^2_U) = \frac{2}{m},$

(2) $R_2(S^2_R) = \frac{1}{m+q}\Big[1 + \frac{\Delta^2(2+\Delta^2)}{m+q}\Big],$

(3) $R_3(\tilde{\sigma}^2_U) = \frac{2}{m+2},$

(4) $R_4(\hat{\sigma}^2_R) = \frac{2}{(m+q+2)}\big[1 + \frac{1}{m+q+2}\Delta^2(2+\Delta^2)\big],$

(5) $R_5(S^2_{\mathrm{PT}[1]}) = E\big(\frac{\chi^2_m}{m} - 1\big)^2 + E\Big[\Big(\frac{1}{m} - \frac{1+\frac{q}{m}F_{q,m}(\Delta^2)}{m+q}\Big)^2 \chi^4_m I\big(F_{q,m}(\Delta^2) < F_{q,m}(\alpha)\big)\Big]$
$-2E\left[\left(\frac{\chi^2_m}{m} - 1\right)\left(\frac{1}{m} - \frac{1+\frac{q}{m}F_{q,m}(\Delta^2)}{m+q}\right)\chi^2_m I\left(F_{q,m}(\Delta^2) < F_{q,m}(\alpha)\right)\right]$
$= \frac{2}{m} + \frac{q^2}{(m+q)^2}E\left[\chi^4_m\left(1 - F_{q,m}(\Delta^2)\right)^2 I\left(F_{q,m}(\Delta^2) < F_{q,m}(\alpha)\right)\right]$
$-\frac{2q}{m+q}E\left[\chi^4_m(1 - F_{q,m}(\Delta^2))I(F_{q,m}(\Delta^2) < F_{q,m}(\alpha))\right]$
$+\frac{2q}{m+q}E\big[\chi^2_m\big(1 - F_{q,m}(\Delta^2)\big)I\big(F_{q,m}(\Delta^2) < F_{q,m}(\alpha)\big)\big]$

(6) $R_6(S^2_{\mathrm{PT}[2]}) = E\Big(\frac{\chi^2_m}{m+2} - 1\Big)^2 + E\left[\Big(\frac{1}{m+2} - \frac{1+\frac{q}{m}F_{q,m}(\Delta^2)}{m+q+2}\Big)^2 \chi^4_m I\big(F_{q,m}(\Delta^2) < F_{q,m}(\alpha)\right]$
$-2E\Big[\big(\frac{\chi^2_m}{m+2} - 1\big)\Big(\frac{1}{m+2} - \frac{1+\frac{q}{m}F_{q,m}(\Delta^2)}{m+q+2}\Big)\chi^2_m I\big(F_{q,m}(\Delta^2) < F_{q,m}(\alpha)\big)\Big]$
$= \frac{2}{m+2} + \frac{q^2}{m^2(m+q+2)^2}E\Big[\big(\frac{m}{m+2} - F_{q,m}(\Delta^2)\big)^2\chi^4_m I\big(F_{q,m}(\Delta^2) < F_{q,m}(\alpha)\big)\Big]$
$-\frac{2q}{m(m+2)(m+q+2)}E\Big[\big(\frac{m}{m+2} - F_{q,m}(\Delta^2)\big)\chi^4_m I\big(F_{q,m}(\Delta^2) < F_{q,m}(\alpha)\big)\Big]$
$+\frac{2q}{m(m+q+2)}E\Big[\big(\frac{m}{m+2} - F_{q,m}(\Delta^2)\big)\chi^2_m I\big(F_{q,m}(\Delta^2) < F_{q,m}(\alpha)\big)\Big].$

(7) $R_7\big(S^2_{[S]}\big) = \frac{2}{m+2} + \frac{q^2}{m^2(m+q+2)^2} E\Big[\big(\frac{m}{m+2} - F_{q,m}(\Delta^2)\big)^2 \chi^4_m I\big(F_{q,m}(\Delta^2) < \frac{m}{m+2}\big)\Big]$

$-\frac{2q}{m(m+2)(m+q+2)} E\Big[\big(\frac{m}{m+2} - F_{q,m}(\Delta^2)\big)^2 \chi^4_m I\big(F_{q,m}(\Delta^2) < \frac{m}{m+2}\big)\Big]$

$+\frac{2q}{m(m+q+2)} E\Big[\big(\frac{m}{m+2} - F_{q,m}(\Delta^2)\big) \chi^2_m I\big(F_{q,m}(\Delta^2) < F_{q,m}\frac{m}{m+2}\big)\Big].$

Note that expressions for $R_7(S^2_{[S]})$ is obtained from $R_6(S^2_{\mathrm{PT}[2]})$ by putting $F_{q,m}(\alpha) = \frac{m}{m+2}$. Actually, this is the optimum value of $F_{q,m}(\alpha)$ for the minimum of $R_6(S^2_{\mathrm{PT}[2]})$ as a function of $F_{q,m}(\alpha)$. Similarly, by Giles (1988) the minimum value of $R_5(S^2_{\mathrm{PT}[1]})$ is obtained at $F_{q,m}(\alpha) = 1$. Hence, the optimum α^* is obtained in these two cases

$$\left.\begin{array}{ll} \alpha^* = F^{-1}_{q,m}\big(\frac{m}{m+2}\big) & \text{for } R_6\big(S^2_{\mathrm{PT}[2]}\big) \\ \alpha^{**} = F^{-1}_{q,m}(1) & \text{for } R_6\big(S^2_{\mathrm{PT}[1]}\big) \end{array}\right\} \text{ for all } (q,m)$$

respectively.

4 Properties of the Estimators

In this section we compare the estimators using the risks criteria. First we note that $\tilde{\sigma}^2_n$ is better than S^2_U and $\hat{\sigma}^2_R$ is better than S^2_R. The PTE $S^2_{\mathrm{PT}[1]}$ is a combination of S^2_U and S^2_R while $S^2_{\mathrm{PT}[2]}$ and $S^2_{[S]}$ are combinations of $\tilde{\sigma}^2_U$ and $\hat{\sigma}^2_R$. The optimum risks of $S^2_{\mathrm{PT}[1]}$ and $S^2_{\mathrm{PT}[2]}$ are obtained when $F_{q,m}(\alpha^{**}) = 1$ and $F_{q,m}(\alpha^{**}) = \frac{m}{m+2}$.

We compare S^2_U and S^2_R. The risk of S^2_U is constant while the risk of S^2_R depends on Δ^2. Under H_0, the risk of $\frac{2}{m+q} < \frac{2}{m}$. Thus, RUE is better than S^2_U under H_0. However, if H_0 does not hold, then the range of Δ^2 for which S^2_R dominates S^2_U is given by the positive root of the following equation

$$\Delta^4 + 2\Delta^2 - \frac{2q}{m} = 0.$$

Let $\Delta^2_* = -1 + \sqrt{1 + \frac{2q}{m}}$ be the positive root of the above equation. Then, S^2_R dominates S^2_U in the interval $[0, \Delta^2_*]$ and S^2_U dominates S^2_R outside this interval. note that as $\Delta^2 \to \infty$, the risk is unbounded while that of S^2_U is constant.

Similarly, compare $\tilde{\sigma}^2_U$ and $\hat{\sigma}^2_R$. In this case $\tilde{\sigma}^2_U$ has constant risk, while the risk of $\hat{\sigma}^2_R$ depends on Δ^2. Thus, under H_0, $\hat{\sigma}^2_R$ dominates $\tilde{\sigma}^2_U$ while in general $\hat{\sigma}^2_R$ dominates $\tilde{\sigma}^2_U$ in the interval $[0, \Delta^2_{**}]$ where Δ^2_{**} is the positive root of the equation

$$\Delta^4 + 2\Delta^2 - \frac{2q}{m+2} = 0.$$

Let $\Delta^2_{**} = -1 \pm \sqrt{1 + \frac{2q}{m+2}}$ be the positive root, then the interval in question is $[0, \Delta^2_{**}]$.

Now we show that $S^2_{[S]}$ dominates $\tilde{\sigma}^2_U$ uniformly under the loss function $\frac{1}{\sigma^4}(\sigma^2_* - \sigma^2)^2$, i.e. $R_7(S^2_{[S]}) \leq R_3(\tilde{\sigma}^2_U)\ \forall\ \Delta^2$.

Consider the risk of $S^2_{[S]}$ with respect to the above loss. Then we have

$$\frac{1}{\sigma^4}E\big[mS_U^2\phi_S(\mathcal{L}_n)-\sigma^2\big]^2=E_{\mathcal{L}_n}\big\{\big[\phi_S^2(\mathcal{L}_n)E(\chi_m^4|\mathcal{L}_n)-2\phi_S(\mathcal{L}_n)E(\chi_m^2|\mathcal{L}_n)+1\big]\big\}$$

where $\phi_S(\mathcal{L}_n)$ is a real function of $\mathcal{L}_n$. The minimum of the quadratic form inside the bracket for fixed Δ^2 and $\mathcal{L}_n$ is given by

$$\phi_S^*(\mathcal{L}_n)=\frac{E(\chi_m^2|\mathcal{L}_n)}{E(\chi_m^4|\mathcal{L}_n)}$$

which is a function of $\mathcal{L}_n$ as well as Δ^2.

Following Stein (1964), it is clear that the maximum of $\phi_S^*(\mathcal{L}_n)$ is attained at $\Delta^2=0$ which eliminates the need of noncentral chi-square distribution. Thus, by straightforward computation we have

$$\phi_0(\mathcal{L}_n)=\frac{E[\chi_m^2/\mathcal{L}_n]}{E[\chi_m^4/\mathcal{L}_n]}=\frac{1+\frac{q}{m}\mathcal{L}_n}{m+q+2}$$

since χ_q^2 and χ_m^2 are independent. If $\mathcal{L}_n<\frac{1}{m+2}$ then $\frac{1+\frac{q}{m}\mathcal{L}_n}{m+q+2}<\frac{1}{m+2}$ which also implies that $\phi^*(\mathcal{L}_n)\leq\phi_0(\mathcal{L}_n)\leq\frac{1}{m+2}\ \forall\ \Delta^2$ that is, $\phi_0(\mathcal{L}_n)$ is closer to the minimizing value than $1/(m+2)$. Thus, defining

$$\phi_S(\mathcal{L}_n)=\frac{1}{m+2}I\Big(\mathcal{L}_n\geq\frac{m}{m+2}\Big)+\frac{1+\frac{q}{m}\mathcal{L}_n}{m+q+2}I\Big(\mathcal{L}_n<\frac{m}{m+2}\Big),$$

we have the Stein-type estimator.

It is then clear from Stein (1964) that

$$E\Big[\Big\{\phi_S(\mathcal{L}_n)\Big(\frac{mS_U^2}{\sigma^2}\Big)-1\Big\}^2\Big|\mathcal{L}_n\Big]\leq E\Big[\Big\{\frac{1}{m+2}\Big(\frac{mS_U^2}{\sigma^2}\Big)-1\Big\}^2\Big|\mathcal{L}_n\Big].$$

Thus $\phi_S(\mathcal{L}_n)mS_U^2$ is better than $mS_U^2/m+2$.

Similarly, we consider the estimator $S^2_{\mathrm{PT}[1]}$ with $F_{q,m}(\alpha)=1$ i.e. $\alpha^{**}=F^{-1}_{q,m}(1)$ for all (q,m) under H_0, then

$$\begin{aligned}{S^*}^2_{\mathrm{PT}[1]}&=\psi^*(\mathcal{L}_n)mS_U^2\\&=S_U^2I(\mathcal{L}_n\geq1)+S_R^2I(\mathcal{L}_n<1).\end{aligned}$$

We show that ${S^*}^2_{\mathrm{PT}[1]}$ dominates S_U^2. In this case too we have

$$\psi_{10}^*(\mathcal{L}_n)=\frac{1+\frac{q}{m}\mathcal{L}_n}{m+q}$$

and the $\max_{\Delta^2}\psi_{10}^*(\mathcal{L}_n)=\frac{1+\frac{q}{m}\mathcal{L}_n}{m+q}$ at $\Delta^2=0$. Then, for $\mathcal{L}_n<1$, we have

$$\frac{1+\frac{q}{m}\mathcal{L}_n}{m+q}<\frac{1}{m}\Longrightarrow\psi_{10}^*(\mathcal{L}_n)\leq\psi_{10}^*(\mathcal{L}_n)\leq\frac{1}{m}\ \forall\ \Delta^2$$

that means $\psi_{10}^*(\mathcal{L}_n)$ is closer to the minimizing value $1/m$. Thus, defining

$$\psi_1(\mathcal{L}_n) = \frac{1}{m} I(\mathcal{L}_n \geq 1) + \frac{1 + \frac{q}{m}\mathcal{L}_n}{m+q} I(\mathcal{L}_n < 1)$$

we have the PTE of σ^2 with 1 as the critical value. Then, it is clear, as in Stein (1964), that

$$E\big[\big(\psi_1(\mathcal{L}_n)\chi_m^2 - 1\big)^2 \big| \mathcal{L}_n\big] \leq E\Big[\big(\frac{1}{m}\chi_m^2 - 1\big)^2 \big| \mathcal{L}_n\Big].$$

Thus, $\psi_1(\mathcal{L}_n) m S_U^2$ is uniformly better than S_U^2 for all Δ^2.

What about the dominance of $\phi_S(\mathcal{L}_n) m S_U^2$ over $\psi_1(\mathcal{L}_n) m S_U^2$? Clearly, $\phi_S(\mathcal{L}_n) \leq \frac{1}{m+2} \leq \psi_1(\mathcal{L}_n) \leq \frac{1}{m}$ which implies that

$$\begin{aligned} E\Big[\big(\phi_S(\mathcal{L}_n)\chi_m^2 - 1\big)^2 \big| \mathcal{L}_n\Big] &\leq E\Big[\Big(\frac{1}{m+2}\chi_m^2 - 1\Big)^2 \big| \mathcal{L}_n\Big] \\ &\leq E\big[\big(\psi(\mathcal{L}_n)\chi_m^2 - 1\big)^2 \big| \mathcal{L}_n\big] \leq E\Big[\Big(\frac{\chi_m^2}{m} - 1\Big)^2 \big| \mathcal{L}_n\Big] \end{aligned}$$

Thus, the Stein-type estimator dominates the PTE with critical value 1. Therefore, we conclude that the Stein-type estimator is the best among the estimators we considered.

Acknowledgements

The author is grateful to Prof A. K. Md. Ehsanes Saleh, Carleton University, Canada for his valuable comments and suggestions on the earlier draft of the manuscript.

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