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A METHOD OF OBTAINING DISTRIBUTIONS OF TRANSFORMED RANDOM VARIABLES BY USING THE HEAVISIDE AND THE DIRAC GENERALIZED FUNCTIONS

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summary

Chi AU and Judy TAM in [1] and Samia WAHED and M.Masoom ALI in [2] compared the method of using the Dirac generalized function [3], [4] with the conventional Change of Variable Technique of transformed continuous random variables for several distributions. In the present article, we have applied the concept of generalized probability density function based on the Heaviside and the Dirac generalized functions for transformed random variables to obtain the distributions of several transformed discrete and continuous random variables. Moreover, we give a theorem for obtaining the joint distribution of a system of transformed continuous random variables and some of its applications.

Keywords and phrases: Distributions of functions of random variables, Generalized function, Heaviside generalized function, Dirac generalized function, Distribution of a system of random variables.

AMS Classification: 62E15

1 Introduction

The importance of the method of finding the probability density function of a function of one or more continuous random variables using the Dirac generalized function [6] was argued in [1], [2], [3] and [4].

The method which uses the Heaviside and the Dirac generalized functions in obtaining distributions and probability density functions of transformed random variables possesses some advantages, compared to the conventional change-of-variable technique.

We note that the method of obtaining distributions of transformed random variables by using the Heaviside and the Dirac generalized functions can simply operates even in the case, when the condition of the existence of a one-to-one transformation between the given variables and transformed variables is not fulfilled. However, the conventional change-of-variable technique operates under the conditions of the existence of a one-to-one transformation and of the computation of the Jacobian, which additionally must be different from zero. It should be noted that limitations of the change of variables technique are discussed in detail in [2].

2 Variables Of Discrete Type

Theorem 1. *Suppose that X_i $(i = 1, ..., n)$ are discrete random variables with joint probability distribution $f(x_1, ..., x_n)$. Let A be n-dimensional set of every possible outcome of the X_i's. Then the random variable*

$$Y = \varphi(X_1, ..., X_n) \tag{2.1}$$

has the probability distribution given by

$$\begin{aligned} G(y) &= P(Y < y) = P(\varphi(x_1, ..., x_n) < y) \\ &= \sum_{(x_1,...,x_n)\in A} f(x_1, ..., x_n) H(y - \varphi(x_1, ..., x_n)), \end{aligned} \tag{2.2}$$

where

$$H(y - \varphi(x_1, ..., x_n)) = \begin{cases} 1, & \text{if } y - \varphi(x_1, ..., x_n) > 0 \\ 0, & \text{if } y - \varphi(x_1, ..., x_n) \leq 0 \end{cases}$$

H-Heaviside generalized function [5].

Theorem 2. *The random variable*

$$Y = \varphi(X_1, ..., X_n)$$

of (2.1) *has the generalized probability density function given by*

$$g(y) = G'(y)$$

$$= \sum_{(x_1,...,x_n)\in A} f(x_1, ..., x_n)H'(y - \varphi(x_1, ..., x_n))$$
$$= \sum_{(x_1,...,x_n)\in A} f(x_1, ..., x_n)\delta(y - \varphi(x_1, ..., x_n)), \tag{2.3}$$

where $\delta(.)$ is the Dirac generalized function.

Remark: The function $G(y)$ is not differentiable in the ordinary sense. But it is differentiable in the sense of generalized functions. Formula (2.3) expresses a generalized function using generalized functions. Therefore we can consider random variables of discrete or continuous types by the same approach.

Example 2.1. *Let X, Y are discrete random variables with joint probability distribution*

$$f(x_i, y_j) = \begin{cases} 0, \text{ if } i + j \neq 7 \; i, j = 1, ..., 6 \\ \frac{1}{6}, \text{ if } i + j = 7 \; x_i = i; \; y_j = j. \end{cases}$$

Find the distribution of the variable $Z = X + Y$.

Solution. By (2.2) the distribution of Z is obtained as following.

$$\begin{aligned} G(z) &= P(Z < z) = P(X + Y < z) \\ &= \sum_{i,j=1}^{6} f(x_i, y_j)H(z - (x_i + y_j)) = \begin{cases} 0, \text{ if } \; z < 7 \\ 1, \text{ if } \; z > 7 \end{cases} \\ &= H(z - 7). \end{aligned}$$

By Theorem 2, we have

$$g(z) = G'(z) = H'(z - 7) = \delta(z - 7).$$

Example 2.2. *Under conditions of example 1, find the distribution of $Z = X^2 + Y$.*

Solution. By (2.2) the distribution of Z is obtained as follows.

$$\begin{aligned} G(z) &= P(Z < z) = P(X^2 + Y < z) \\ &= \sum_{i,j=1}^{6} f(x_i, \; y_j)H(z - (x_i^2 + y_j)) \\ &= \frac{1}{6}H(z - 7) + \frac{1}{6}H(z - 9) + \frac{1}{6}H(z - 13) + \frac{1}{6}H(z - 19) \\ &\quad + \frac{1}{6}H(z - 27) + \frac{1}{6}H(z - 37). \end{aligned}$$

By Theorem 2, we have

$$
\begin{aligned}
g(z) &= G'(z) = \frac{1}{6}\{\delta(z-7)+\delta(z-9)+\delta(z-13)+\delta(z-19) \\
&\quad +\delta(z-27)+\delta(z-37)\}.
\end{aligned}
$$

3 Variables of Continuous Type

Theorem 3. *Suppose that X_i $(i = 1, ..., n)$ are continuous random variables with joint probability density $f(x_1, ..., x_n)$. Let A be the n-dimensional set of every possible outcome of the X_i's. Then the random variable*

$$
Y = \varphi(X_1, ..., X_n) \tag{3.1}
$$

has the probability distribution function given by

$$
\begin{aligned}
G(y) &= P(Y < y) = P(\varphi(x_1, ..., x_n) < y) \\
&= \int \underset{D(y)}{\cdots} \int f(x_1, \ldots, x_n) \times \; dx_1 \ldots dx_n \\
&= \int \underset{A}{\cdots} \int f(x_1, \ldots, x_n) \times \; H(y - \varphi(x_1, \ldots, x_n)) \; dx_1 \ldots dx_{,n}
\end{aligned} \tag{3.2}
$$

where $D(y) = \{(x_1, ..., x_n) \in A : \varphi(x_1, ..., x_n) < y\}$, and H is as defined in (2.2).

Theorem 4. *The random variable*

$$
Y = \varphi(X_1, ..., X_n)
$$

of (3.1) *has the probability density function given by*

$$
\begin{aligned}
g(y) &= G'(y) \\
&= \int \underset{A}{\cdots} \int f(x_1, \ldots, x_n) \times \delta(y - \varphi(x_1, \ldots, x_n)) dx_1 \ldots dx_n.
\end{aligned} \tag{3.3}
$$

Proof. The relationship $H'(t) = \delta(t)$ exists between the Heaviside and the Dirac generalized functions. Consequently,

$$
H'(y - \varphi(x_1, ..., x_n)) = \delta(y - \varphi(x_1, ..., x_n))
$$

and (3.3) follows from the formula (3.2). □

We note that Theorem 4 is a corollary of Theorem 3 and coincides with Theorem 2 of [1].

Example 3.1. *Let X be a continuous random variable with probability density function $f(x)$, $x \in [a, \infty)$, and $y = \varphi(x)$ be a differentiable monotone function. Find the distribution of $Y = \varphi(X)$.*

Solution (By definition of distribution). We have

$$G(y) = P(Y < y) = P(\varphi(x) < y) = \int_{D(y)} f(x)\, dx,$$

where $D(y) = \{x : \varphi(x) < y,\ x \in (a, \infty)\}$.Since $\varphi(.)$ is monotone, then $D(y) = \{x : \psi(y) \wedge \psi(a) < x < \psi(y) \vee \psi(a)\}$, where $\psi(.)$ is inverse function of $\varphi(.)$.

Then

$$g(y) = G'(y) = f(\psi(y)) \left|\psi'(y)\right|.$$

By Theorem 4, we have

$$g(y) = \int_a^\infty f(s)\delta(\varphi(s) - y)ds,$$

where

$$\delta(\varphi(s) - y) = \frac{1}{|\varphi'(s)|}\delta(s) = \frac{1}{\frac{1}{|\psi'(y)|}}\delta(s)$$

$$= \left|\psi'(y)\right| \delta(s).$$

Consequently,

$$g(y) = \int_a^\infty f(s) \left|\psi'(y)\right| \delta(y)ds = \left|\psi'(y)\right| f(\psi(y)).$$

Example 3.2. Let X_1 and X_2 are two independently distributed random variables, where $X_1 \backsim Gamma(\alpha, 1)$ and $X_2 \backsim Gamma(\beta, 1)$ with densities f_{X_1}, and f_{X_2}. Show that $Y = X_1 + X_2$ has a $Gamma(\alpha + \beta, 1)$ distribution.

Solution (By using definition of distribution).

Note that

$$G(y) = \int_0^\infty dx_1 \int_0^{y-x_1} \frac{x_1^{\alpha-1}\ x_2^{\beta-1} e^{-(x_1+x_2)}}{\Gamma(\alpha)\Gamma(\beta)} dx_2.$$

Then,

$$g(y) = G'(y) = \frac{1}{\Gamma(\alpha)\Gamma(\beta)} \int\limits_0^\infty x_1^{\alpha-1} e^{-x_1} (y - x_1)^{\beta-1} e^{-(y-x_1)} dx_1$$

$$= \frac{e^{-y}y^{\beta-1}}{\Gamma(\alpha)\Gamma(\beta)} \int_0^\infty x_1^{\alpha-1}(1-\frac{x_1}{y})^{\beta-1}dx_1.$$

By substitution $\frac{x_1}{y}=t$, we get

$$g(y) = \frac{e^{-y}y^{\beta-1}}{\Gamma(\alpha)\Gamma(\beta)} \int_0^1 (yt)^{\alpha-1}(1-t)^{\beta-1}ydt,$$

or

$$g(y) = \frac{e^{-y}y^{\alpha+\beta-1}}{\Gamma(\alpha+\beta)}.$$

Solution (By Theorem 4).

By applying formula (3.3) we have

$$g(y) = \int_0^\infty\int_0^\infty \frac{1}{\Gamma(\alpha)\Gamma(\beta)} x_1^{\alpha-1}x_2^{\beta-1}e^{-(x_1+x_2)}\delta((x_2+x_1)-y)dx_1dx_2.$$

By

$$\delta(x_1+x_2-y) = \delta(x_2-(y-x_1))$$

we get

$$g(y) = \frac{1}{\Gamma(\alpha)\Gamma(\beta)} \int_0^\infty x_1^{\alpha-1}e^{-x_1}\left\{\int_0^\infty x_2^{\beta-1}e^{-x_2}\delta(x_2-(y-x_1))dx_2\right\}dx_1$$

$$\begin{aligned} &= \frac{1}{\Gamma(\alpha)\Gamma(\beta)} \int_0^\infty x_1^{\alpha-1}e^{-x_1} \times \left\{(y-x_1)^{\beta-1}e^{-(y-x_1)}\right\}dx_1 \\ &= \frac{e^{-y}}{\Gamma(\alpha)\Gamma(\beta)} \int_0^\infty x_1^{\alpha-1}(y-x_1)^{\beta-1}dx_1 = \frac{y^{\alpha+\beta-1}e^{-y}}{\Gamma(\alpha+\beta)}. \end{aligned}$$

Solution (By using change of variable technique).

Define

$$\begin{aligned} Y_1 &= X_1+X_2,\ x_1 = y_1y_2 \\ Y_2 &= \frac{X_1}{X_1+X_2},\ x_2 = y_1(1-y_2). \end{aligned}$$

Then the Jacobian is

$$J = -y_1.$$

Consequently,

$$\begin{aligned} g(y_1, y_2) &= |y_1| \frac{1}{\Gamma(\alpha)\Gamma(\beta)} (y_1 y_2)^{\alpha-1} \times (y_1(1-y_2))^{\beta-1} e^{-y_1} \\ &= \frac{y_2^{\alpha-1}(1-y_2)^{\beta-1}}{\Gamma(\alpha)\Gamma(\beta)} y_1^{\alpha+\beta-1} e^{-y_1}, 0 < y_1, y_2 < \infty. \end{aligned}$$

Hence

$$g(y_1) = \int_0^1 g(y_1, y_2) dy_2$$

$$\begin{aligned} &= \frac{y_1^{\alpha+\beta-1} e^{-y}}{\Gamma(\alpha)\Gamma(\beta)} \int_0^1 y_2^{\alpha-1}(1-y_2)^{\beta-1} dy_2, \\ &= \frac{y_1^{\alpha+\beta-1} e^{-y_1}}{\Gamma(\alpha+\beta)}. \end{aligned}$$

Next we shall give some essential auxiliary facts to present a generalization of Theorem 4 for system of transformed random variables. Let be

$$H(\mathbf{x}) = \prod_{i=1}^{n} H(x_i), \quad \mathbf{x} = (x_1, ..., x_n); \; H(x_i) = \begin{cases} 1, & \text{if } x_i > 0 \\ 0, & \text{if } x_i \leq 0 \end{cases},$$

then this function defines a linear functional called the Heaviside generalized function on the set of test functions:

$$(H, g) = \int_{E_n} H(\mathbf{x}) g(\mathbf{x}) d\mathbf{x} = \int_{-\infty}^{\infty} ... \int_{-\infty}^{\infty} g(x_1, ..., x_n) dx_1 ... dx_2,$$

where g is arbitrary test function.

If

$$\delta(\mathbf{x}) = \prod_{i=1}^{n} \delta(x_i), \quad \mathbf{x} = (x_1, ..., x_n);$$

then in the sense of generalized functions the equality

$$\frac{\partial^n H}{\partial y_1 ... \partial y_n} = \delta(\mathbf{x}), \quad \mathbf{x} = (x_1, ..., x_n) \tag{3.4}$$

holds.

The formula (3.4) means that for an arbitrary test function g, we have

$$\begin{aligned}(H, \frac{\partial^n g}{\partial x_1 ... \partial x_n}) &= \int_{E_n} H(\mathbf{x}) \frac{\partial^n g}{\partial x_1 ... \partial x_n} dx_1 ... dx_2 = (-1)^n \varphi(0, ..., 0) \\ &= (-1)^n \int_{E_n} H(\mathbf{x}) g(\mathbf{x}) d\mathbf{x};\end{aligned}$$

$$\delta(\varphi(\mathbf{x})) = \sum_{k=1}^{m} \frac{1}{\left| \frac{D(\varphi_1, ..., \varphi_n)}{D(x_1, ..., x_n)} \right|} \delta(\mathbf{x} - \mathbf{x}^{(k)}),$$

where $\mathbf{x}^k \in E_n$, $k = 1, ..., m$ such that $\varphi_i(\mathbf{x}^k) = 0,\ i = 1, ..., n;$ $\frac{D(\varphi_1, ..., \varphi_n)}{D(x_1, ..., x_n)} |_{\mathbf{x}=\mathbf{x}^k} \neq 0.$

The formula (3.4) also means a generalized function. That is, if $f(\mathbf{x})$ is a continuous function, $\mathbf{x} \in E_n$, then following equality holds:

$$\int_{E_n} f(\mathbf{x}) \delta(\mathbf{x}) d\mathbf{x} = \sum_{k=1}^{m} f(x^k) \frac{1}{\frac{D(\varphi_1, ..., \varphi_n)}{D(x_1, ..., x_n)} |_{\mathbf{x}=\mathbf{x}^k}}.$$

A generalization of Theorem 4 for system of transformed random variables is following.

Theorem 5. *Let* $\mathbf{X} = (X_1, ..., X_n)$ *be continuous random vector with joint probability density function* $f(\mathbf{x})$, $\mathbf{x} \in A$, $A \subset E_n$, *where* A *is the* n*-dimensional set of every possible outcome of the* $\mathbf{X}'s$. *Then the random variable*

$$\mathbf{Y} = \varphi(\mathbf{X}),\ \mathbf{Y} = (Y_1, ..., Y_r),\ r \leq n$$

has the generalized probability density function given by

$$g(\mathbf{y}) = \int_A f(\mathbf{x}) \delta(\mathbf{y} - \varphi(\mathbf{x})) dx, \tag{3.5}$$

where δ *is the Dirac generalized* n*-dimensional function:*

$$\delta(\mathbf{x}) = \prod_{i=1}^{n} \delta(x_i),\ \mathbf{x} = (x_1, ..., x_n).$$

Proof: The distribution function of $\mathbf{Y}$ is given by

$$G(\mathbf{y}) = P(\mathbf{Y} < \mathbf{y}) = \int_{D(\mathbf{y})} f(\mathbf{x}) d\mathbf{x},$$

where

$$D(\mathbf{y}) = \{\mathbf{x} \in A:\ \varphi(\mathbf{x}) < \mathbf{y}\},$$

or

$$G(\mathbf{y}) = P(\mathbf{Y} < \mathbf{y}) = \int\limits_A f(\mathbf{x})H(\mathbf{y} - \varphi(\mathbf{x}))d\mathbf{x}.$$

Then the random variable $\mathbf{Y} = \varphi(\mathbf{x})$, $Y = (Y_1, ..., Y_r)$ has the generalized p.d.f. given by

$$\begin{aligned} g(\mathbf{y}) &= \frac{\partial^r G(\mathbf{y})}{\partial y_1 ... \partial y_r} = \int\limits_A f(\mathbf{x}) \frac{\partial^r H(\mathbf{y} - \varphi(\mathbf{x}))}{\partial y_1 ... \partial y_r} d\mathbf{x}, \\ &= \int\limits_A f(\mathbf{x}) \prod_{i=1}^{r} H'(y_i - \varphi_i(\mathbf{x}))d\mathbf{x} \\ &= \int\limits_A f(\mathbf{x})\delta(\mathbf{y} - \varphi(\mathbf{x}))d\mathbf{x}, \end{aligned}$$

or

$$g(\mathbf{y}) = \int\limits_A f(\mathbf{x})\delta(\mathbf{y} - \varphi(\mathbf{x}))d\mathbf{x}.$$

Theorem 5 is proved.

Theorem 5 indicates that if the number of transformed variables is not equal to the number of the given variables then the method of obtaining distributions of transformed vector variables by using Dirac generalized n-dimensional functions operates successfully.

The following example is an application of Theorem 5.

Example 3.3. Let $X_1,\ X_2,\ X_3$ be independently distributed random variables with joint Chi-square density function

$$f_{X_i}(x_i) = \frac{x_i^{\frac{1}{2}} e^{-\frac{x_i}{2}}}{\Gamma(\frac{1}{2})2^{\frac{1}{2}}}.$$

Let

$$Y_1 = \varphi_1(X_1, X_2) = \frac{X_1}{X_2},\ Y_2 = \varphi_2(X_1, X_2) = \frac{X_3}{X_1 + X_2}.$$

Find the joint probability density function of (Y_1, Y_2).

Solution: In order to apply Theorem 5, we have to solve the system

$$y_1, = \frac{x_1}{x_2};\quad y_2 = \frac{x_3}{x_1 + x_2}$$

with respect to x_1 and x_2, to get

$$x_1 = \frac{y_1 x_3}{(y_1 + 1)y_2},\quad x_2 = \frac{x_3}{(y_1 + 1)y_2}.$$

The Jacobian of latter system is $\dfrac{-x_3^2}{(y_1 + 1)^2 y_2^3}$.

By virtue of Theorem 5 the joint probability density function of (Y_1, Y_2) is given by:

$$\begin{aligned} f(y_1,y_2) &= \int_0^\infty\int_0^\infty\int_0^\infty f(\mathbf{x})\delta(\mathbf{y}-\varphi)d\mathbf{x} \\ &= \int_0^\infty\int_0^\infty\int_0^\infty f(x_1,x_2,x_3)\ \delta(y_1-\varphi_1)\ \delta(y_2-\varphi_2)\ dx_1dx_2dx_3 \\ &= \int_0^\infty\int_0^\infty\int_0^\infty f_{X_1}(x_1)\ f_{X_2}(x_2)\ f_{X_3}(x_3)\ \delta(y_1-\varphi_1)\delta(y_2-\varphi_2)dx_1dx_2dx_3 \\ &= \int_0^\infty f_{X_1}(\frac{y_1x_3}{(y_1+1)y_2})\ f_{X_2}(\frac{x_3}{(y_1+1)y_2})\times\left|\frac{-x_3^2}{(y_1+1)^2y_2^3}\right|dx_3 \end{aligned}$$

$$\begin{aligned} &= \int_0^\infty \frac{(\frac{y_1x_3}{(y_1+1)y_2})^{\frac{1}{2}}e_i^{-\frac{y_1x_3}{2(y_1+1)y_2}}\ (\frac{x_3}{(y_1+1)y_2})^{\frac{1}{2}}e^{-\frac{x_3}{2(y_1+1)y_2}}x_3^{\frac{1}{2}}e^{\frac{x_3}{2}}}{\Gamma(\frac{1}{2})^3 2^{\frac{3}{2}}} \\ &\quad\times\left|\frac{-x_3^2}{(y_1+1)^2y_2^3}\right|dx_3 \\ &= \frac{2^3\ \Gamma(\frac{9}{2})\ y_1^{\frac{1}{2}}\ y_2^{\frac{1}{2}}}{\Gamma(\frac{1}{2})^3\ (1+y_1)^3(1+y_2)^{9/2}}, \end{aligned}$$

or

$$f(y_1,y_2) = \frac{2^3\ \Gamma(\frac{9}{2})\ y_1^{\frac{1}{2}}\ y_2^{\frac{1}{2}}}{\Gamma(\frac{1}{2})^3\ (1+y_1)^3(1+y_2)^{9/2}},\ y_1>0,\ y_2>0.$$

The following example is solved by using another method in [7].

Example 3.4. (see [7]) Let $(X\ ,Y)$ be random vector variable with joint probability density function:

$$f_{XY}(x,y) = \frac{1}{2\pi\sigma^2}e^{-\frac{(x^2+y^2)}{2\sigma^2}}.$$

Find the probability density function $g(z,w)$ of vector variable (Z,W), where

$$Z=\sqrt{X^2+Y^2};\quad W=\frac{Y}{X}. \tag{3.6}$$

Solution.

$$\frac{D(x,y)}{D(z,w)} = \frac{\sqrt{x^2+y^2}}{x^2},$$

and the system (3.6) with respect to $x,\ y$ has the following solutions

$$x^{(k)} = \pm\frac{z}{\sqrt{1+w^2}};\ y^{(k)} = \pm\frac{wz}{\sqrt{1+w^2}},\quad k=1,2.$$

This means that between $x,\ y$ and $z,\ w$ a one-to-one transformation does not exist. By using Theorem 5, we get

$$\begin{aligned} g(z,w) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\delta(z-\sqrt{x^2+y^2})\delta(w-\frac{y}{x})dx\ dy \\ &= \sum_{k=1}^{2} f(x^{(k)},y^{(k)}) \times \left|\frac{x^2}{\sqrt{x^2+y^2}}\right| \\ &= \frac{z}{1+w^2}\{f(\frac{z}{\sqrt{1+w^2}},\frac{wz}{\sqrt{1+w^2}}) + f(-\frac{z}{\sqrt{1+w^2}},\frac{wz}{\sqrt{1+w^2}})\}, \end{aligned}$$

$$g(z,w) = \frac{ze^{-\dfrac{z^2}{2\sigma^2}}}{\pi\sigma^2(1+w^2)}.$$

This example shows that the method generalized by Dirac n-dimensional generalized function (Theorem 5) operates simply even in the case, when a one-to-one transformation between the given random variables and transformed random variables does not exist. In this case conventional change-of-variable technique doesn't work.

4 Discussion

This article presents an approach to determine probability distributions of the functions of random variables using the Heaviside and the Dirac generalized functions. On the basis of this approach, a generalized probability density function concept is defined and the fact that both discrete and continuous random variables, in the sense of generalized functions, have the same nature is showed. Selecting an effective mathematical approach allows us to similarly examine problems and concepts which seem to be different. The concept of generalized probability density function for different random variables serves in accordance with this objective to certain extent.

The method using the Heaviside and the Dirac generalized functions have important advantages compared to conventional change-of-variable technique. The mentioned advantages are showed on examples. However, we should be careful about that each technique has some advantages and disadvantages for special encountered problems.

The expression of the distribution of discrete random variables by the aid of the Heaviside function is given in Theorem 1 and the expression of the density function of the discrete random variables by the aid of the Dirac function is given in Theorem 2.

Theorem 3 and Theorem 4 express analogical statements belonging to continuous random variables.

By Theorem 5 distributions of system of transformed random variables are obtained when the number of transformed variables is not equal to the number of the given variables and an one-to-one transformation between the given variables and transformed variables is not fulfilled.

The method given by Theorem 5 operates simply even in the case, when the condition of the existence of a one-to-one transformation between the given random variables and transformed variables is not fulfilled.

Class of random variables that is investigated for the distributions is significantly expanded by the concept of generalized density function. Therefore, the concept of generalized density function may give some contributions in some fields of theoretical statistics.

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