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CONDITIONAL INTERVAL ESTIMATION FOR THE RATIO OF SHAPE PARAMETERS IN POWER LAW PROCESS

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SUMMARY

Power law process (PLP) or Weibull process is used to model reliability growth and is usually characterized by the intensity function $\lambda(x)$. We investigate the conditional confidence interval (CCI) for the ratio of two PLP shape parameters following the rejection of the hypothesis $H_0 : \lambda_1(x) = \lambda_2(x)$. This interval has been found to be possibly shorter than the unconditional confidence interval (UCI). The conditional coverage probability (CCP) of the confidence interval is obtained by computing the coverage probability under the conditional probability density function. The CCP of the UCI is not uniformly greater than or less than the nominal level. The UCI is appropriate only when one does not perform any preliminary test (pre-test). However, if a pre-test is performed before the construction of a confidence interval, then the appropriate interval is the CCI as the pre-test affects subsequent inference procedures. The method is illustrated on a data set.

Keywords and phrases: Intensity function, Confidence interval, Preliminary test, Maximum likelihood, Repairable systems, Shape parameter, Sufficient statistics.

AMS Classification: Primary 62F25, Secondary 62E18.

1 Introduction

Let $T_1 < T_2 < \cdots < T_n$ be the first n occurrence times of a random point process with $T_0 = 0$. Let N_t be the number of events occurred between 0 and t. This process $\{N_t; t \geq 0\}$ was first introduced by Duane (1964) and is defined by the intensity function

$$\lambda(t) = \left(\frac{\beta}{\theta}\right)\left(\frac{t}{\theta}\right)^{\beta-1} \tag{1.1}$$

Such a model is referred to as the power law process (PLP). The process parameters θ and β are respectively called the scale and shape parameter of the process. The process described in (1.1) is also called non-homogeneous Poisson process (NHPP) with mean value function $m(t) = \left(\frac{t}{\theta}\right)^{\beta}$. The expected rate of occurrence is $\frac{d}{dt}m(t)$, which is nothing but (1.1). For $\beta = 1$ the process reduces to a homogeneous Poisson Process (HPP). Otherwise, a PLP provides a model for a system whose reliability changes as it ages. If $\beta > 1$, it models a deteriorating system and when $\beta < 1$, it provides a model for reliability growth. This process is sometimes called Weibull process as the intensity is the power of time.

Most of the inferences on PLP are done on two counts. They are inferences based on time truncated data and failure truncated data sets. Crow (1974, 1982), Bain and Engelhardt (1980), Ascher and Fiengold (1984), Bain et al. (1985), Rigdon and Basu (1989, 2000) have discussed various inferences of this model under time truncated data. Lee and Lee (1978), Bain and Engelhardt (1991), Baker (1996), Black and Rigdon (1996), Jani et al. (1997), Muralidharan (1999), and Gaudoin et al. (2006) have provided tests of hypothesis and other inferences based on failure truncated data. Gaudoin (1998) has given goodness-of-fit tests for the PLP based on conditional probability integral transformations. Recently, Muralidharan (2002 a, b) has studied reliability inferences of Weibull process and modulated power law process under different realizations.

Soland (1969), Calabria and Pulcini (1990), Lingham and Sivaganean (1997, 1999) have studied the Bayesian analysis of the Weibull process with unknown scale and shape parameter. For the problems related to predictive inferences we refer to Bar-Lev et al. (1992) and Muralidharan et al. (2006).

For repairable systems when an NHPP is assumed to be the model, the intensity function solely describes the probability features of the process. Hence, comparing such a repairable system with a standard one requires comparing their corresponding intensity functions. Hence a test for $H_0 : \lambda_1(x) = \lambda_2(x)$ becomes important to assess the trend in the process. If the intensity functions of the two processes are proportional, then it would mean that the observed system is aging in the same manner as the standard one (see also Bhatacharjee et al. 2004). If the ratio of the two intensity function shows an increasing trend, then it means that failures (consequently repairs) are becoming more frequent for the process as compared with the standard system. Hence in the sense of frequency of failures, the new system is deteriorating relative to the standard system. Hence to distinguish between the above two, test for H_0 is warranted. Since the failure processes are orderly and T_i' s are positive, the above test can be equivalently stated as testing $H_0 : \beta_1 = \beta_2$ against $H_0 : \beta_1 \neq \beta_2$ or any suitable one sided alternative.

There are instances in life testing experiments where estimation following a preliminary test of hypothesis on a parameter of interest has been employed suitably to obtain confidence limits. Such inferences are done using conditional inferences. It has also been shown that such confidence intervals are found to be possibly shorter in length compared to the length of the unconditional confidence intervals. For an early references see Bancroft (1944), Bancroft and Han (1977), Kiefer (1977), Robinson (1979), Lehmann (1986 p. 558), Han et al. (1988) etc. Conditional confidence intervals for the mean of a normal distribution following rejection of a preliminary hypothesis have been examined by Meeks and D'Agostino (1983). Conditional confidence intervals for the exponential location parameter, correlation coefficient of the bi-variate normal distribution and ratio of extreme-value shape parameter have been examined by Chiou and Han (1995), Chiou (2000) and by Chiou and Chang (2004) respectively.

The objective of this study is to provide the conditional confidence interval for the ratio of two PLP shape parameters following the rejection of the hypothesis $H_0 : \beta_1 = \beta_2$. In Section 2 some important results and properties of the process are discussed. The conditional confidence interval is provided in Section 3. Some simulation study and examples are discussed in the subsequent Sections.

2 Some Useful Results

Let $t_1, t_2, \ldots, t_n$ denote the first n successive times of occurrences from (1.1). Then the joint probability density function of $t_1, t_2, \ldots, t_n$ as

$$f(t_1, t_2, \ldots, t_n) = \left(\frac{\beta}{\theta}\right)^n \left(\prod_{i=1}^{n} \frac{t_i}{\theta}\right)^{\beta-1} e^{-\left(\frac{t_n}{\theta}\right)^\beta}, \tag{2.1}$$

$0 < t_1 < t_2 < \cdots < t_n < \infty$, $\beta > 0, \theta > 0$. Further, $\frac{f(t_1,t_2,\ldots,t_n)}{f(u_1,u_2,\ldots,u_n)} = k(t,u)$, for every $(\theta, \beta) \in \Omega$, iff $t_n = u_n$ and $\sum_{i=1}^{n-1} \log(t_i) = \sum_{i=1}^{n-1} \log(u_i)$, which establishes the fact that $\left(t_n, \sum_{i=1}^{n-1} \log(t_i)\right)$ is jointly sufficient statistic for (θ, β). For known β, T_n is the complete sufficient statistic for θ. Also

$$f(t_i) = \int_{t_1} \int_{t_2} \int_{t_3} \cdots \int_{t_{i-1}} \int_{t_{i+1}} \cdots \int_{t_n} f(t_1, t_2, \ldots, t_n)$$

$$dt_1 dt_2 \ldots dt_{i-1} dt_{i+1} \ldots dt_n = \frac{\beta}{\theta^{i\beta}\Gamma(i)} t_i^{i\beta-1} e^{-\left(\frac{t_i}{\theta}\right)^\beta},\ 0 < t_i < \infty$$

and

$$f(t_n) = \frac{\beta}{\theta^{n\beta}\Gamma(n)} t_n^{n\beta-1} e^{-\left(\frac{t_n}{\theta}\right)^\beta}, t_n > 0.$$

So that the conditional distribution of $(t_1, t_2, \ldots, t_{n-1})$ given t_n is

$$f(t_1, t_2, \ldots, t_{n-1}|t_n) = \Gamma(n)\left(\frac{\beta}{t_n}\right)^{n-1}\prod_{i=1}^{n}\left(\frac{t_i}{t_n}\right)^{\beta-1}, \; 0 < t_1 < t_2 < \cdots < t_n \tag{2.2}$$

The density (2.2) is free from the nuisance parameter θ. The following results also hold for this process.

Result 1.

If $u_i = (t_i/t_n)^{\beta}$, then $u_1, u_2, \ldots, u_{n-1}$ are order statistics from $U(0,1)$ and $u_n = 1$.

Result 2.

The maximum likelihood estimates (MLE) of θ and β are $\hat{\theta} = t_n/n^{1/\hat{\beta}}$ and $\hat{\beta} = n/\sum_{i=1}^{n-1}\ln\left(\frac{t_n}{t_i}\right)$ respectively.

Result 3.

If $S = \sum_{i=1}^{n-1}\ln(t_n/t_i)$, then $2\beta S$ has chi-square distribution with $2(n-1)$ degrees of freedom.

Result 4.

To test $H_0 : \beta = \beta_0$ against $H_0 : \beta \neq \beta_0$ a uniformly most powerful unbiased test (UMPU) is to reject H_0 if $V(x) \notin (c_1, c_2)$, where $V(x) = \sum_{i=1}^{n-1}\log\left(\frac{t_i}{t_n}\right)$ and for large n, $c_1 = \frac{1}{\beta_0}\chi^2_{(2(n-1),\alpha/2)}$ and $c_2 = \frac{1}{\beta_0}\chi^2_{(2(n-1),1-\alpha/2)}$.

The result 4 is available in Muralidharan (1999) and is derived based on the conditional distribution given in (2.2). This conditional test also coincides with that of Bain and Engelhardt (1980). The conditional confidence interval is constructed in the next section.

3 Conditional Confidence Interval

Exploiting the idea of conditional inference, we now obtain the conditional confidence interval (CCI) for $\phi = \frac{\beta_1}{\beta_2}$ upon rejection of the test $H_0 : \beta_1 = \beta_2$ against $H_0 : \beta_1 \neq \beta_2$. Let $f_{\alpha/2}$ and $f_{1-\alpha/2}$ denote the $100(\alpha/2)$ and $100(1-\alpha/2)$ percentage points of the F distribution with $[2(n_1-1), 2(n_2-1)]$ degrees of freedom, then an α level critical region of the test $H_0 : \beta_1 = \beta_2$ against $H_0 : \beta_1 \neq \beta_2$ is defined by

$$\Psi(x) = \begin{cases} 1, & \text{if } T < f_{\alpha/2} \quad \text{or} \quad T > f_{1-\alpha/2} \\ 0, & \text{otherwise,} \end{cases} \tag{3.1}$$

where $T = \frac{(n_2-1)S_1}{(n_1-1)S_2}$ and α is such that $P_{r_{H_0}}[T < f_{\alpha/2} \text{ or } T > f_{1-\alpha/2}] = \alpha$. Since $\frac{\beta_1}{\beta_2}T$ has an F distribution with $[2(n_1-1), 2(n_2-1)]$ degrees of freedom, the usual $100(1-p_1-p_2)\%$ unconditional confidence interval (UCI) for ϕ is $\frac{f_{p_1}}{T} \leq \phi \leq \frac{f_{1-p_2}}{T}$, where p_1 and p_2 are the lower and upper tail probabilities. For constructing UCI we need not have a preliminary test. The power of the test say $\beta(\phi)$ is obtained a

$$\beta(\phi) = Pr_{H_1}\left[T < f_{\alpha/2} \text{ or } T > f_{1-\alpha/2}\right]$$

$$= Pr_{H_1}\left[T\left(\frac{\beta_1}{\beta_2}\right) < f_{\alpha/2}\left(\frac{\beta_1}{\beta_2}\right) \text{ or } T\left(\frac{\beta_1}{\beta_2}\right) > f_{1-\alpha/2}\left(\frac{\beta_1}{\beta_2}\right)\right]$$

$$= 1 + F\left(\phi f_{\alpha/2}\right) - F\left(\phi f_{1-\alpha/2}\right). \tag{3.2}$$

The conditional confidence interval is constructed based on the conditional distribution of T given the power of the test. If $f(t)$ denote the unconditional pdf of T, then the conditional pdf of T is obtained as

$$f_c(t) = \begin{cases} \frac{f(t)}{\beta(\phi)}, & \text{if } \ t < f_{\alpha/2} \ \text{ or } \ t > f_{1-\alpha/2} \\ 0, & \text{otherwise.} \end{cases} \tag{3.3}$$

Note that $f_c(t)$ is defined only when $\beta(\phi) > 0$ and for $\alpha > 0$, $f_c(t)$ is always defined. For ϕ other than unity (say 0 or ∞) and α very large (say $\alpha \to 1$) the conditional density converges to the unconditional pdf. In Figure 1, we present the graph of $F_c(t)$ for various values of ϕ. It is observed that for $\phi < 1$, $F_c(t)$ is increasing and for $\phi > 1$, $F_c(t)$ is decreasing. We will use this fact to compute the conditional confidence interval in the following theorem.

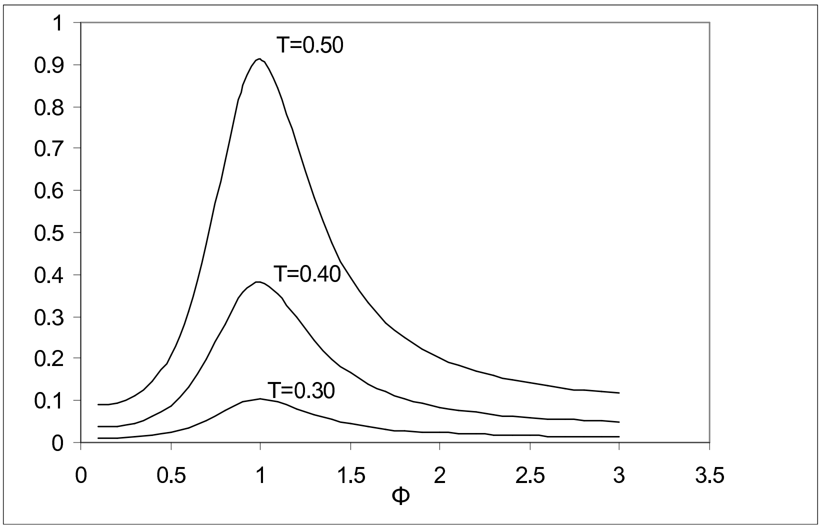

Figure 1: The graph of $F_c(t)$

Theorem 1. *Let* $\beta_i(\phi) = 1 + F\left(\phi_i^c\, f_{\alpha/2}\right) - F\left(\phi_i^c\, f_{1-\alpha/2}\right)$, $i = L, U$.

1. *If* $p_1 < \frac{1}{\beta_L(\phi)} F\left(\phi_L^c\, f_{\alpha/2}\right)$ *and* $1 - p_2 \leq \frac{1}{\beta_U(\phi)} F\left(\phi_U^c\, f_{\alpha/2}\right)$, *then the solutions of the equations*
$$p_1 = \frac{1}{\beta_L(\phi)} F\left(T\, \phi_L^c\right) \tag{3.4}$$
and
$$1 - p_2 = \frac{1}{\beta_U(\phi)} F\left(T\, \phi_U^c\right) \tag{3.5}$$
constitutes a $100(1 - p_1 - p_2)\%$ *confidence interval for* ϕ

2. *If* $p_1 < \frac{1}{\beta_L(\phi)} F\left(\phi_L^c\, f_{\alpha/2}\right)$ *and* $1 - p_2 > \frac{1}{\beta_U(\phi)} F\left(\phi_U^c\, f_{\alpha/2}\right)$, *then the solutions of the equations (3.4) and equation*
$$1 - p_2 = \frac{1}{\beta_U(\phi)} \left(F\left(T\, \phi_U^c\right) + F\left(\phi_U^c\, f_{\alpha/2}\right) - F\left(\phi_U^c\, f_{1-\alpha/2}\right)\right) \tag{3.6}$$
constitutes a $100(1 - p_1 - p_2)\%$ *confidence interval for* ϕ.

3. *If* $p_1 \geq \frac{1}{\beta_L(\phi)} F\left(\phi_L^c\, f_{\alpha/2}\right)$ *and* $1 - p_2 > \frac{1}{\beta_U(\phi)} F\left(\phi_U^c\, f_{\alpha/2}\right)$, *then the solutions of the equations (3.6) and equation*
$$p_1 = \frac{1}{\beta_L(\phi)} \left(F\left(T\, \phi_L^c\right) + F\left(\phi_L^c\, f_{\alpha/2}\right) - F\left(\phi_L^c\, f_{1-\alpha/2}\right)\right) \tag{3.7}$$
constitutes a $100(1 - p_1 - p_2)\%$ *confidence interval for* ϕ.

The theorem is a direct consequence of results (3.2) and (3.3). For given values of T, α, p_1, p_2, the limits ϕ_L^c and ϕ_U^c can be obtained by solving the equations in the above theorem. The ratio of the length of the CCI to that of UCI is given by $R = \left(\phi_U^c - \phi_L^c\right) / \left(\frac{f_{1-p_2}}{T} - \frac{f_{p_1}}{T}\right)$. Table 1 provides the value of the ratio of length of a 90% CCI to the length of a 90% UCI for $n_1 = 10$, $n_2 = 10$, $\alpha = 0.10$ and for given values of T. Note that the ratio does not exist when $f_{\alpha/2} \leq T \leq f_{1-\alpha/2}$. When T approaches $f_{\alpha/2}(= 0.4510)$ from the left, the length of the CCI can be much smaller than the length of the UCI. As T increases to infinity or decreases to zero, the ratio exceeds one and eventually converges to one from above. This can be illustrated in the following way:

Consider $n_1 = n_2 = 10, \alpha = 0.01$ and $p_1 = p_2 = 0.05$. Then $f_{\alpha/2} = 0.4510$ and $f_{1-\alpha/2} = 2.2172$. If the observed value of T is 0.40, then the hypothesis is rejected as $T < f_{\alpha/2}$. The UCI for ϕ is $[1.12375, 5.5429]$ while the CCI is $[0.7231, 3.4760]$ obtained by solving the nonlinear equations in Theorem 1. Although the CCI is shorter than UCI, it contains the value of $\phi = 1$. It usually happens if the value of T happens to be close to $f_{\alpha/2}$. The length of the ratio of CCI to that of UCI is 0.622948 which is smaller than unity. If T is 0.25 then the UCI and CCI are respectively obtained as $[1.8041, 8.8688]$ and $[1.6354, 9.1245]$. The length of the ratio of CCI to that of UCI is 1.060073 which exceeds one. Here CCI is slightly larger than the UCI and does not contain the value of ϕ. A similar behavior is

Table 1: ► *Ratio of length of CCI to UCI*

T	R	T	R	T	R	T	R
0.01	1.801	0.25	0.998	2.5	0.223	5	0.976
0.05	1.785	0.3	0.983	3	0.478	10	1.004
0.1	1.583	0.35	0.727	3.5	0.735	20	1.063
0.15	1.405	0.4	0.357	4	0.856	30	1.078
0.2	1.02	0.45	0.035	4.5	0.923	40	1.089

observed when the value of T is larger. This establishes the fact that as ϕ goes to zero or infinity, the CCI goes to UCI.

To study the actual coverage probability that is provided at the nominal $100(1-p)\%$ level we compute the coverage probability of the UCI under the conditional pdf of T. We also obtain the power curve in Figure 2. From the graph of $\beta(\phi)$ it is seen that the power increases as ϕ tends to 0 or ∞.

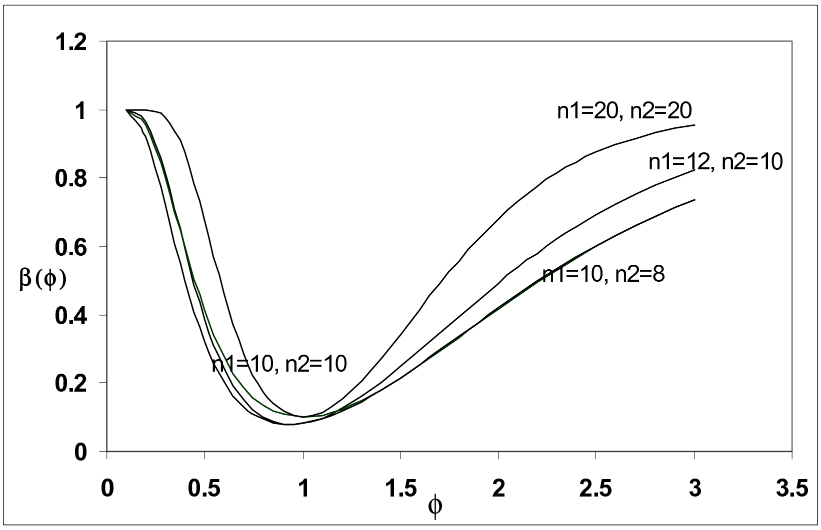

Figure 2: Power of the test for various values of (n_1, n_2) and ϕ

Theorem 2. *1. If $\phi\, f_{\alpha/2} \le f_{p/2}$ and $\phi\, f_{1-\alpha/2} \ge f_{1-p/2}$, then the CCP for ϕ is zero.*

2. If $\phi\, f_{\alpha/2} \le f_{p/2}$ and $f_{p/2} < \phi\, f_{1-\alpha/2} < f_{1-p/2}$, then the CCP for ϕ is

$$\frac{1}{\beta(\phi)}\left[\left(1-\frac{p}{2}\right) - F\left(\phi\, f_{1-\alpha/2}\right)\right].$$

3. *If* $f_{p/2} < \phi\, f_{\alpha/2} < f_{1-p/2}$ *and* $f_{p/2} < \phi\, f_{1-\alpha/2} < f_{1-p/2}$*, then the CCP for* ϕ *is*

$$\frac{1}{\beta(\phi)}\left[(1-p) + F\left(\phi\, f_{\alpha/2}\right) - F\left(\phi\, f_{1-\alpha/2}\right)\right].$$

4. *If* $f_{p/2} < \phi\, f_{\alpha/2} < f_{1-p/2}$ *and* $\phi\, f_{1-\alpha/2} \geq f_{1-p/2}$*, then the CCP for* ϕ *is*

$$\frac{1}{\beta(\phi)}\left[F\left(\phi\, f_{\alpha/2}\right) - \frac{p}{2}\right].$$

5. *If* $\phi\, f_{\alpha/2} > f_{1-p/2}$ *or* $\phi\, f_{1-\alpha/2} < f_{p/2}$*, then the CCP for* ϕ *is*

$$\frac{1}{\beta(\phi)}\left(1-p\right).$$

Table 2 presents the nominal $100(1-p)\%$ coverage probability of the UCI for $n_1 = 20, n_2 = 15, p = 0.10$ and for various values of ϕ and α. The CCP of the UCI has a maximum value of 0.945695 when $\alpha = 0.1$ that is about 5% higher than the nominal level 0.90, however, it can be as low as 0 at $\phi = 1$ that is far less than the nominal level. On the other hand, if a larger value of α is used, say 0.5, then it has a maximum value of higher than 0.967856 and a minimum value of 0.776567 at $\phi = 1$ which is not much different from the UCI due to the fact that CCI→UCI as $\alpha \to 1$. Evidently, ϕ is unknown in practice, and a decision to use the UCI after rejection of H_0 is inappropriate as the pre-test affects any subsequent inference procedure.

4 Numerical Example

We now present two examples. The first example is based on Kumar and Klefsjo (1992) data on hydraulic systems of LHD machines. The authors have considered the failure data of six different machines: two each of old (LHD 1 and LHD 3), medium (LHD 9 and LHD 11) and new (LHD 17 and LHD 20). They have used PLP models for checking the presence of trends in the time between failures of the hydraulic system.

According to them the null hypothesis of equality of two shape parameters is accepted in every case when two different systems are compared. Although their conclusion is same everywhere, they have obtained the value of test statistic using $T = \frac{\hat{\beta}_1}{\hat{\beta}_2}$ instead of T as defined above. Since the hypothesis is accepted in every case the CCI can not be constructed. The estimates and UCI for ϕ when two the shape parameters are tested are presented in Table 3.

The second example is the failure data from 1978 to 1998 in steel pipelines split in to three sets according to the different types of corrosion which caused them namely, natural, galvanic and by stray currents. We reproduce the data from Ruggeri (2006) as follows:

Table 2: ▶ *Conditional Coverage Probability of UCI*

ϕ	$\alpha = 0.05$	$\alpha = 0.10$	$\alpha = 0.20$	$\alpha = 0.40$	$\alpha = 0.50$
0.1	0.907047	0.902673	0.900822	0.900182	0.900034
0.2	0.941472	0.945695	0.933196	0.910766	0.900235
0.3	0.915146	0.930677	0.940227	0.945244	0.923768
0.4	0.859235	0.898994	0.922124	0.932916	0.916576
0.5	0.750834	0.839796	0.886637	0.904265	0.902365
0.6	0.546975	0.731783	0.819266	0.84684	0.895644
0.7	0.184487	0.542631	0.698736	0.78368	0.854278
0.8	0.1213	0.256254	0.513283	0.751567	0.756785
0.9	0.0000	0.1567	0.427775	0.733468	0.789456
1.0	0.0000	0.0000	0.429422	0.732691	0.776567
1.1	0.0000	0.291832	0.481541	0.745145	0.798998
1.2	0.129789	0.481888	0.550707	0.76404	0.865432
1.3	0.35792	0.611659	0.730987	0.784143	0.898654
1.4	0.510417	0.698333	0.790877	0.802716	0.823675
1.5	0.614178	0.757227	0.83099	0.818777	0.843545
1.6	0.686888	0.798463	0.858518	0.877425	0.887875
1.7	0.739403	0.828268	0.877973	0.895353	0.904535
1.8	0.778397	0.850451	0.892135	0.908179	0.900235
1.9	0.808061	0.867393	0.90273	0.917513	0.903467
2.0	0.831107	0.880622	0.910851	0.924422	0.923676
2.1	0.849336	0.891148	0.917209	0.929619	0.954355
2.2	0.86398	0.899661	0.92228	0.933588	0.945345
2.3	0.8759	0.90664	0.926387	0.936662	0.946758
2.4	0.88571	0.912431	0.929758	0.939072	0.953456
2.5	0.893857	0.917283	0.932559	0.940984	0.957643
2.6	0.900675	0.921383	0.934908	0.942518	0.945678
2.7	0.906411	0.924873	0.936896	0.943759	0.943567
2.8	0.911257	0.927858	0.938591	0.944772	0.946786
2.9	0.91536	0.930424	0.940044	0.945606	0.956787
3	0.918836	0.932634	0.941298	0.946298	0.967856

Table 3: ► *UCI for hydraulic systems of LHD machines*

Machine	*Estimate*		*LHD Comparison*	*Statistic T*	*UCI*
	β	θ			
LHD 1	1.628	363.9	(1 vs 3)	0.9204	(0.6660, 1.7384)
LHD 3	1.493	408			
LHD 9	1.654	646.6	(9 vs 11)	0.7967	(0.7969, 1.9679)
LHD 11	1.316	231.7			
LHD 17	1.528	383.2	(17 vs 20)	0.7943	(0.7779, 2.0553)
LHD 20	1.22	253.2			

Galvanic : 2.1233, 3.5205, 4.3945, 8.9041

Natural : 2.8438, 4.1534, 7.2383, 9.5232, 9.8082, 9.819, 9.8219, 12.4931, 13.8904, 14.4136, 15.7890, 16.1013

Stray Currents : 0.0027, 0.1041, 0.3507, 1.1753, 3.9726, 5.0320, 5.2932, 5.7616, 7.0219, 11.7425, 11.7616, 15.3918, 16.164

The author has concluded NHPP models for galvanic and natural corrosion data and HPP for the third case under Bayesian set up. The parameter estimates of (θ, β) for galvanic, natural and stray currents are (3.0752, 1.3039), (4.1588, 1.8356) and (0.0878, 0.4918) respectively. It is interesting to compare the shape parameters of galvanic and natural type of corrosion with that of stray currents type. It is shown that the hypothesis $H_0 : \beta_{gal} = \beta_{str}$ is accepted as the T value is 0.46421 which is in between $f_{\alpha/2}(= 0.2603)$ and $f_{\alpha_{1-\alpha/2}}(= 2.5082)$. The UCI for $\phi\left(= \frac{\beta_{gal}}{\beta_{str}}\right)$ is [0.560776, 5.40313]. When the hypothesis is tested for natural corrosion against stray currents, the T value obtained is 0.26979 which is less than $f_{\alpha/2}(= 0.4930)$ and hence the hypothesis $H_0 : \beta_{nat} = \beta_{str}$ is rejected for $\alpha = 0.10$. In this case, the UCI interval for $\phi\left(= \frac{\beta_{nat}}{\beta_{str}}\right)$ is [1.8274, 7.4281], the corresponding CCI is [1.5847, 6.7960] and the length ratio is 0.9307.

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