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ESTIMATOR OF MEAN IN AN INVERSE GAUSSIAN POPULATION BASED ON THE COEFFICIENT OF VARIATION

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summary

Srivastava (1974, 1980) and Chaubey and Dwivedi (1982) investigated some estimators of mean of a normal population utilizing an estimate of the coefficient of variation. However, the normal model may not hold for positive or positively skewed data, hence an alternative model may have to be employed. This paper uses the inverse Gaussian model for such data and extends the results of Chaubey and Dwivedi (1982) for the normal population to similar analysis for the inverse Gaussian population. It is found that the new estimator may result in large gains in efficiency over the sample mean for large values of the coefficient of variation.

Keywords and phrases: Inverse Gaussian population; Relative bias; Relative mean square error.

AMS Classification: 62F10

1 Introduction

Searles (1964) considered estimating a population mean when the coefficient of variation (*CV*) is known. Note that the *CV* for a population with mean μ and standard deviation σ is given by

$$CV = \sqrt{\sigma^2/\mu^2}.$$

He showed that the sample mean $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$, based on a random sample $X_1, X_2, ..., X_n$, can be improved in the sense of reduced mean square error (*MSE*) by considering the class of estimators $\mathcal{G}_c = \{c\bar{X}, c > 0\}$. It can be easily seen that the constant c which minimizes the $MSE(c\bar{X})$ is given by

$$c = 1 + \frac{\phi}{n}, \tag{1.1}$$

where

$$\phi = (CV)^2.$$

Thus the minimum mean square error estimator of μ in the class $\mathcal{G}_c$ is given by

$$\tilde{\mu} = \frac{1}{1 + \frac{\phi}{n}} \bar{X}. \tag{1.2}$$

This improved estimator requires the exact value of the coefficient of variation or equivalently the value of ϕ which may be unknown in practice. In such a situation, Srivastava (1974) and Thompson (1968) proposed estimators similar to that given in (1.2), but ϕ replaced by

$$\hat{\phi} = \frac{S^2}{\bar{X}^2},$$

where S^2 is the sample variance, considered as an estimator of the unknown variance σ^2. This leads to the estimator,

$$\hat{\mu} = \frac{\bar{X}}{1 + \frac{S^2}{n\bar{X}^2}} \tag{1.3}$$

which, however, loses the optimal property of the estimator in (1.1). Srivastava (1980) studied the finite sample bias and MSE properties of $\hat{\mu}$, for the case of a normal population, where as Chaubey and Dwivedi (1982) studied similar properties of a modified estimator

$$\tilde{\mu}_k = \frac{\bar{X}}{1 + k\frac{\hat{\phi}}{n}}, \tag{1.4}$$

where k is a non-negative constant to be suitably chosen. Note that the estimator given in (1.3) is a special case of the above estimator with $k = 1$.

Ideally, k should be chosen so that $MSE(\tilde{\mu}_k)$ is minimum. However, as we will see in the next section, the expression for $MSE(\tilde{\mu}_k)$ is not of simple form and, therefore the solution for k which minimizes $MSE(\tilde{\mu}_k)$ is not readily available. We may propose a numerical solution, however, this too is not feasible because the expression for $MSE(\tilde{\mu}_k)$ still involves the unknown value of ϕ. To avoid this problem, Chaubey and Dwivedi (1982) consider the value of k which may be motivated by considering kS^2 as an estimator of σ^2. They investigated three choices of k, namely, (i) $k = (n-1)/(n+1)$ which gives the minimum mean square error estimator of σ^2, (ii) $k = 1$, for the unbiased estimator of σ^2 and (iii) $k = (n-1)/(n-3)$, for the mode estimator of σ^2 (see Höglund (1974)) in order to assess the gain in efficiency by using the modified estimator over the sample mean. Note that the modified estimator for the choices (i) and (ii) is defined for $n \geq 2$ where as that for the choice (iii), we need a sample of size at least 4.

The aim of the present article is to extend the work of Chaubey and Dwivedi (1982) for the normal population to similar analysis for the inverse Gaussian population. The inverse Gaussian distribution with parameters μ and λ, denoted by $IG(\mu, \lambda)$ is described by the probability density function

$$f(x; \mu, \lambda) = \left(\frac{\lambda}{2\pi x^3}\right)^{1/2} \exp\left(-\frac{\lambda(x-\mu)^2}{2\mu^2 x}\right), \quad 0 < x < \infty. \tag{1.5}$$

This distribution was extensively studied by Tweedie (1957a, b) but it is popularized by the review article by Folks and Chhikara (1978). It is now widely used for modeling positive and/or positively skewed data in such diverse areas of applied research as cardiology, hydrology, demography, linguistics, employment service, labor disputes, finance, reliability and life testing; see Chhikara and Folks (1989) and Seshadri (1999). The parameter μ describes the mean and λ describes the dispersion as the variance of the distribution is given by

$$\sigma^2 = \frac{\mu^3}{\lambda}.$$

For a random sample $X_1, X_2, ..., X_n$ from $IG(\mu, \lambda)$ population, a minimal sufficient statistic for (μ, λ) is given by $(\bar{X}, \sum_{i=1}^n \frac{1}{X_i})$. It is also interesting to note that

$$\bar{X} \sim IG(\mu, n\lambda) \quad \text{and} \quad \lambda \sum_{i=1}^n \left(\frac{1}{X_i} - \frac{1}{\bar{X}}\right) \sim \chi^2_{n-1} \tag{1.6}$$

and moreover they are independent. As such, $\bar{X}$ provides the minimum variance unbiased estimator of μ and

$$U = \frac{1}{n-1} \sum_{i=1}^n \left(\frac{1}{X_i} - \frac{1}{\bar{X}}\right) \tag{1.7}$$

provides that for $\frac{1}{\lambda}$. The square of the coefficient of variation ϕ, in this case, is given by

$$\phi = \frac{(\mu^3/\lambda)}{\mu^2} = \frac{\mu}{\lambda}.$$

Assuming that the parameters (μ, λ), are not known, the estimators of μ and λ, as given above may be used to estimate the squared CV, ϕ which is given by

$$\hat{\phi} = \bar{X}U. \tag{1.8}$$

Using the above estimator in (1.4), we propose to investigate the bias and MSE properties of the estimator

$$\hat{\mu}_k = \frac{\bar{X}}{1 + k\frac{\bar{X}U}{n}}. \tag{1.9}$$

Chaubey and Dwivedi (1982) provided two types of analytical moment-results for the estimator in (1.4) for the normal population, one in terms of a series expansion and the other in terms of an univariate integral. Similar results can be obtained for the $IG(\mu, \lambda)$ populations. The series expansion is of limited utility though, because it is applicable only for odd sample sizes and may require a large number of terms for adequate computational accuracy. Hence, we omit the details of this approach and refer the reader to Sen (2004). Alternatively, we develop an expression in the form of a univariate integral, which is found to be more appropriate for computational purpose as given Section 2. The final section presents a comparison of the new estimator with the sample mean for various choices of the sample size and ϕ with three choices of the constant k, such that kU provides some well known estimator of $1/\lambda$: (i) $k = (n-1)/n$ for the MLE, (ii) $k = 1$ for the unbiased estimator and (iii) $k = (n-1)/(n-3)$ for the mode estimator.

2 An Univariate Integral Representation for $E(\hat{\mu}_k^r)$

The following theorem provides an integral representation for the the r^{th} moment of $\hat{\mu}_k$ given in (1.9).

Theorem 1. *The r^{th} raw moment of $\hat{\mu}_k$ is given by*

$$E(\hat{\mu}_k^r) = \frac{\mu^r}{\Gamma(r)(2\tau)^r} \int_0^1 g_r(w)dw, \tag{2.1}$$

where

$$g_r(w) = w^{r-1}(1-w)^{\frac{\nu}{2}-r-1} e^{\frac{n}{\phi}[1-\left(1+\frac{w\nu}{k(1-w)}\right)^{\frac{1}{2}}]} \left(1 + \frac{w\nu}{k(1-w)}\right)^{-\frac{1}{2}},$$
$$\tau = \frac{k\phi}{n(n-1)}, \quad and \quad \nu = n-1.$$

Proof. Write $\hat{\mu}_k$ as

$$\hat{\mu}_k = \frac{\mu Z}{1 + \frac{k\phi Z V}{n(n-1)}} \tag{2.2}$$

where

$$Z = \frac{\bar{X}}{\mu} \sim IG(1, \frac{n}{\phi}) \text{ and } V = (n-1)\lambda U \sim \chi^2_{n-1}, V \overset{ind}{\sim} Z.$$

Therefore,

$$\frac{\hat{\mu}_k}{\mu} = \frac{Z_1}{Z_2} \tag{2.3}$$

where

$$Z_1 = Z, Z_2 = 1 + \tau Z V \text{ and } \tau = \frac{k\phi}{n(n-1)}.$$

The moment generating function of (Z_1, Z_2) is, therefore, given by

$$\begin{aligned}\varphi(\theta_1,\theta_2) &= E\left[e^{\theta_1 Z_1+\theta_2 Z_2}\right] \\ &= E\left[e^{\theta_1 Z+\theta_2+\tau\theta_2 ZV}\right] \\ &= e^{\theta_2} E\left[e^{\theta_1 Z+\tau\theta_2 ZV}\right] \\ &= e^{\theta_2} E_Z[e^{\theta_1 Z} E_{V|Z}(e^{\tau\theta_2 ZV})].\end{aligned}$$

Since, the moment generating function of a χ^2_ν random variable is $M_{\chi^2_\nu}(t) = (1-2t)^{-\frac{\nu}{2}}$, and $V \sim \chi^2_\nu, V \overset{ind}{\sim} Z$, by putting $t = 2\tau\theta_2 Z$, the above equation becomes

$$\varphi(\theta_1,\theta_2) = e^{\theta_2} E[e^{\theta_1 Z}(1-2\tau\theta_2 Z)^{-\frac{\nu}{2}}] \tag{2.4}$$

Now we use the following lemma from Chaubey and Dwivedi (1982).

Lemma 2.1. (Chaubey and Dwivedi, 1982) *Let* $Z_2 > 0$ *almost everywhere and*

$$\varphi(\theta_1,\theta_2) = E[\exp(\theta_1 Z_1 + \theta_2 Z_2)]$$

be the joint moment generating function of (Z_1, Z_2), *then*

$$E\left[\left(\frac{Z_1}{Z_2}\right)^r\right] = \frac{1}{\Gamma(r)}\int_{-\infty}^0 (-\theta_2)^{r-1}\frac{\partial^r \varphi(\theta_1,\theta_2)}{\partial\theta_1^r}\,|_{\theta_1=0}\, d\theta_2.$$

Thus, we have

$$\begin{aligned}E\left[\left(\frac{\hat{\mu}_k}{\mu}\right)^r\right] &= E\left(\frac{Z_1}{Z_2}\right)^r \\ &= \frac{1}{\Gamma(r)}\int_{-\infty}^0 (-\theta_2)^{r-1}\frac{\partial^r \varphi(\theta_1,\theta_2)}{\partial\theta_1^r}\,|_{\theta_1=0}\, d\theta_2.\end{aligned} \tag{2.5}$$

But,

$$\frac{\partial^r \varphi(\theta_1,\theta_2)}{\partial\theta_1^r}\,|_{\theta_1=0} = e^{\theta_2} E[Z^r(1-2\tau\theta_2 Z)^{-\frac{\nu}{2}})].$$

Therefore (2.5) becomes

$$\begin{aligned}E\left[\left(\frac{\hat{\mu}_k}{\mu}\right)^r\right] &= \frac{1}{\Gamma(r)}\int_{-\infty}^0 (-\theta_2)^{r-1} e^{\theta_2} E_Z[Z^r(1-\tau\theta_2 Z)^{-\frac{\nu}{2}}]d\theta_2 \\ &= \frac{1}{\Gamma(r)}\int_0^\infty \theta_2^{r-1} e^{-\theta_2} E_Z[Z^r(1+2\tau\theta_2 Z)^{-\frac{\nu}{2}}]d\theta_2 \\ &= \frac{1}{\Gamma(r)}\int_0^\infty \theta_2^{r-1} e^{-\theta_2}\int_0^\infty z^r(1+2\tau\theta_2 z)^{-\frac{\nu}{2}} f(z;1,\frac{n}{\phi})dz d\theta_2.\end{aligned}$$

Now, using the transformation $\theta_2 \to y = 2\tau\theta_2 z, d\theta_2 = \frac{dy}{2\tau z}$ gives

$$\begin{aligned} E\left[\left(\frac{\hat{\mu}_k}{\mu}\right)^r\right] &= \frac{1}{\Gamma(r)}\int_0^\infty\int_0^\infty e^{-\frac{y}{2\tau z}}\left(\frac{y}{2\tau z}\right)^{r-1} z^r(1+y)^{-\frac{\nu}{2}} f(z;1,\frac{n}{\phi})\frac{1}{2\tau z}dydz \\ &= \frac{1}{\Gamma(r)(2\tau)^r}\int_0^\infty\int_0^\infty e^{-\frac{y}{2\tau z}} y^{r-1}(1+y)^{-\frac{\nu}{2}} f(z;1,\frac{n}{\phi})dydz \\ &= \frac{1}{\Gamma(r)(2\tau)^r}\int_0^\infty y^{r-1}(1+y)^{-\frac{\nu}{2}}\left[\int_0^\infty e^{-\frac{y}{2\tau z}} f(z;1,\frac{n}{\phi})dz\right]dy \qquad (2.6)\end{aligned}$$

Noting that the inner integral in the above equation is the Laplace transform of Z^{-1} where $Z \sim IG(1, n/\phi)$, we use the result from Seshadri (1998, formula on the bottom of pp. 51) and get

$$\int_0^\infty e^{-\frac{y}{2\tau z}} f(z;1,\frac{n}{\phi})dz = e^{\frac{n}{\phi}[1-(1+\frac{y\phi}{\tau n})^{\frac{1}{2}}]}(1+\frac{y\phi}{\tau n})^{-\frac{1}{2}}.$$

Putting the above result in (2.6) we get

$$E\left[\left(\frac{\hat{\mu}_k}{\mu}\right)^r\right] = \frac{1}{\Gamma(r)(2\tau)^r}\int_0^\infty y^{r-1}(1+y)^{-\frac{\nu}{2}} e^{\frac{n}{\phi}[1-(1+\frac{y\phi}{\tau n})^{\frac{1}{2}}]}(1+\frac{y\phi}{\tau n})^{-\frac{1}{2}}dy.$$

Substituting $y = \frac{w}{1-w}, dy = \frac{dw}{(1-w)^2}$, into the integral in the above equation we get

$$E\left[\left(\frac{\hat{\mu}_k}{\mu}\right)^r\right] = \frac{1}{\Gamma(r)(2\tau)^r}\int_0^1 g_r(w)dw, \qquad (2.7)$$

where $g_r(w) = w^{r-1}(1-w)^{\frac{\nu}{2}-r-1}e^{\frac{n}{\phi}[1-\left(1+\frac{w\nu}{k(1-w)}\right)^{\frac{1}{2}}]}\left(1+\frac{w\nu}{k(1-w)}\right)^{-\frac{1}{2}}$. This completes the proof of Theorem 1.

□

3 Computations and Comparisons

We wish to evaluate the estimator proposed here in terms of its bias and mean square error. The criteria for comparison used are absolute relative bias (ARB) and relative mean square error (RMSE) as given below,

$$ARB(\hat{\mu}_k) = \frac{E(\hat{\mu}_k) - \mu}{\mu} = \frac{E(\hat{\mu}_k)}{\mu} - 1, \qquad (3.1)$$

$$RMSE(\hat{\mu}_k) = \frac{MSE(\hat{\mu}_k)}{\mu^2} = \frac{E(\hat{\mu}_k^2)}{\mu^2} - 2\frac{E(\hat{\mu}_k)}{\mu} + 1. \qquad (3.2)$$

In order to get a quick idea to judge the superiority of the modified estimator or the lack thereof over the sample mean, we also calculate the relative efficiency (RE) of $\hat{\mu}_k$ as given by

$$RE(\hat{\mu}_k) = \frac{MSE(\bar{X})}{MSE(\hat{\mu}_k)} = \frac{\phi}{n\ RMSE(\hat{\mu}_k)}. \qquad (3.3)$$

Table 1: Absolute Relative Bias of $\hat{\mu}_k$

n	20	40	60	80	100
ϕ		$k = \frac{n-1}{n}$			
00.01	0.000475	0.000247	0.000164	0.000123	0.000099
00.05	0.002375	0.001218	0.000819	0.000617	0.000495
00.10	0.004749	0.002437	0.001639	0.001234	0.000990
01.00	0.047167	0.024331	0.016375	0.012338	0.009897
05.00	0.216125	0.118380	0.080823	0.061228	0.049244
10.00	0.370855	0.222431	0.156437	0.120039	0.097181
		$k = 1.0$			
00.01	0.000500	0.000250	0.000167	0.000125	0.000010
00.05	0.002499	0.001250	0.000833	0.000625	0.000500
00.10	0.004997	0.002500	0.001666	0.001250	0.000100
01.00	0.049509	0.024938	0.016648	0.012492	0.009996
05.00	0.224472	0.121000	0.082070	0.061951	0.049715
10.00	0.381690	0.226641	0.158613	0.121353	0.098058
		$k = \frac{n-1}{n-3}$			
00.01	0.000559	0.000264	0.000173	0.000128	0.000102
00.05	0.002792	0.001317	0.000863	0.000641	0.000510
00.10	0.005581	0.002634	0.001725	0.001282	0.001021
01.00	0.054969	0.026248	0.017222	0.012812	0.010200
05.00	0.243308	0.126606	0.084682	0.063451	0.050685
10.00	0.405545	0.235568	0.163152	0.124071	0.099858

Table 2: Relative MSE of $\hat{\mu}_k$

n	20	40	60	80	100
ϕ		$k = \frac{n-1}{n}$			
00.01	0.000499	0.000250	0.000167	0.000125	0.000100
00.05	0.002482	0.001245	0.000831	0.000624	0.000499
00.10	0.004930	0.002482	0.001659	0.001245	0.000997
01.00	0.043543	0.023254	0.015871	0.012047	0.009708
05.00	0.142442	0.088980	0.065685	0.052102	0.043166
10.00	0.221893	0.139672	0.107486	0.088371	0.075206
		$k = 1.0$			
00.01	0.000499	0.000250	0.000167	0.000125	0.000100
00.05	0.002482	0.001245	0.000831	0.000624	0.000499
00.10	0.004928	0.002482	0.001658	0.001245	0.000997
01.00	0.043386	0.023229	0.015863	0.012044	0.009706
05.00	0.142524	0.088741	0.065569	0.052042	0.043132
10.00	0.225276	0.139727	0.107332	0.088248	0.075120
		$k = \frac{n-1}{n-3}$			
00.01	0.000499	0.000250	0.000167	0.000125	0.000100
00.05	0.002481	0.001245	0.000831	0.000624	0.000499
00.10	0.004924	0.002481	0.001658	0.001245	0.000997
01.00	0.04308	0.023177	0.015847	0.012036	0.009703
05.00	0.143666	0.088306	0.065342	0.051922	0.043063
10.00	0.234337	0.140065	0.107064	0.088011	0.074952

Table 3: Relative Efficiency of $\hat{\mu}_k$

n	20	40	60	80	100
ϕ		$k = \frac{n-1}{n}$			
00.01	1.001402	1.000725	1.000489	1.000369	1.000296
00.05	1.007030	1.003633	1.002448	1.001846	1.001481
00.10	1.014108	1.007279	1.004902	1.003695	1.002965
01.00	1.148291	1.075090	1.050119	1.037589	1.030066
05.00	1.755097	1.404807	1.268681	1.199567	1.158314
10.00	2.253337	1.789904	1.550584	1.414484	1.329690
		$k = 1.0$			
00.01	1.001448	1.000737	1.000494	1.000372	1.000298
00.05	1.007260	1.003693	1.002475	1.001861	1.001491
00.10	1.014567	1.007399	1.004956	1.003726	1.002985
01.00	1.152456	1.076263	1.050655	1.037894	1.030262
05.00	1.754094	1.408589	1.270919	1.200949	1.159237
10.00	2.219502	1.789205	1.552812	1.416460	1.331200
		$k = \frac{n-1}{n-3}$			
00.01	1.001546	1.000762	1.000506	1.000378	1.000302
00.05	1.007746	1.003817	1.002530	1.001892	1.001511
00.10	1.015533	1.007647	1.005067	1.003788	1.003024
01.00	1.160700	1.078663	1.051747	1.038513	1.030660
05.00	1.740152	1.415531	1.275343	1.203718	1.161093
10.00	2.133683	1.784889	1.556698	1.420272	1.334182

The values of k considered here are those motivated by different estimators of λ, namely, (i) $k = \frac{(n-1)}{n}$, (ii) $k = 1$ and (iii) $k = \frac{(n-1)}{(n-3)}$ as described earlier. We have used R-package (R Development Core Team, 2005) and computed the values of RB, RMSE and RE for values of n ranging from 4 to 100 and for values $\phi = .01, .05, .10, 1.0, 5.0, 10$ and 50. A selection of these values is reported in Table 1 for ARB, in Table 2 for RMSE and in Table 3 for RE for three choices of k listed above. For a convenient exposition we also present the RE curve for the values of $\phi = 0.1, 0.5, 1, 5, 10$ and 50.

The integral in (2.7) is evaluated using a modification of recursive Simpson's rule which may be described as follows. Suppose that we have to evaluate the integral of a function $g(x)$ over the interval (a, b). We may evaluate the required integral using the trapezoidal-rule and Simpson's rule respectively. Let these be given by a_1 and a_2 respectively, we report the value a_2 as the required value of the integral if the absolute difference $|a_1 - a_2|$ is less than a prescribed error. If the prescribed error is not reached, we may divide the interval (a, b) into subintervals (a, d) and (d, b) where $d = (a + b)/2$ is the midpoint of the interval (a, b) and apply the same procedure to evaluate the area of these subintervals. The R-codes for this function is obtained from Venebles and Ripley (1994, pp. 107).

For computing the relative bias, we note that $\lim_{w\to 1} g_1(w) = 0$, but $g_1(0) = 1$ and the algorithm described above proposes no problem. However, for computing the relative mean square error, we note that $\lim_{w\to 0} g_2(w) = \lim_{w\to 1} g_2(w) = 0$, and for large values of n, the function $g_2(w)$ decreases to zero very fast, so that $g_2(0.5)$ is close to zero. Thus direct use of the above algorithm terminates quickly with the resulting value zero. To avoid this problem we evaluate the integral of $g_2(w)$ over two intervals, (a, w_0) and (w_0, b), using the above algorithm where w_0 is the approximate value of the argument where the function $g(w)$ peaks. The approximate peak is obtained by taking the maximum of $g(w)$ evaluated over a grid of $w-$values. For the values reported here, an error bound of 10^{-10} is used and that $w_0 = .0001$ was found to be adequate.

Based on the graphs and tables, we draw the following conclusions which are similar to those in Chaubey and Dwivedi (1982) for a Gaussian population. Note also that for smaller values of ϕ, RE values are very close to 1, hence we have not included such small values of ϕ in the graph. The bottom three curves corresponding to the values of $\phi = 0.1, 0.5$ and 1.0 are almost indistinguishable. We see from Figures 1, 2 and 3 that larger values of ϕ are associated with higher gains in efficiency, especially for smaller sample sizes; see the curves for $\phi = 5, 10$ and 50 and sample sizes less than 10 where RE values are larger than 2.0. We may suspect that these efficiencies may be accompanied by large biases. This is apparently true; for example ARB values for $k = 1, \phi = 5$ and sample sizes $n = 4, 5, 7, 10$ are respectively, $0.559127, 0.522103, 0.453208, 0.372111$ and the corresponding RE values are $2.823905, 2.612404, 2.355433, 2.141829$.

We may also make some other general observations based on these tables and figures.

(1) For the choice of k considered here, the ARB values increase as the CV increases for fixed sample size and they decrease with increasing sample size for fixed CV.

(2) Relative mean square error decreases as n increases for fixed CV and it decreases with

increasing CV for a fixed sample size.

(3) There is a positive gain in efficiency of $\hat{\mu}_k$ over $\bar{X}$ for the values of k considered here over the whole range of the CV and sample size. For small values of CV *e.g.* $\phi < 1$), there may not be any noticeable gain in efficiency by using the modified estimator over the sample mean, specially for sample sizes such as $n < 20$. Substantial gains in efficiency are achieved for small samples with large coefficient of variation. However, such large gains may be associated with relatively large biases. Hence attention must be paid on the amount of bias in specific situations.

(4) We see that the gain in relative efficiency is always positive. However, it may be substantial for $\phi \geq 0.1$. Hence, we may wish to use $\hat{\mu}_k$ if a statistical test rejects the hypothesis $H_0 : \phi \leq 0.11$ vs. $H_1 : \phi > 0.1$ This type of problem comes under the area of preliminary test estimator which will be pursued elsewhere.

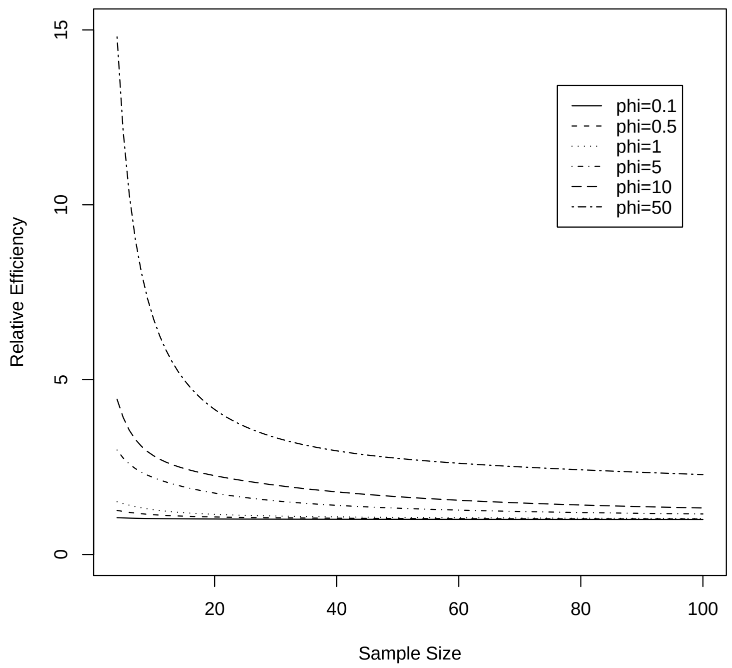

Figure 1: Relative Efficiency of $\hat{\mu}_k, k = \frac{n-1}{n}$

Since the moments of the similar estimator have been studied for the normal case (Srivastava (1980), Chaubey and Dwivedi (1982)) it might be instructive to compare $\tilde{\mu}_k$ and $\hat{\mu}_k$ with respect to bias and MSE properties. Note that while $\tilde{\mu}_k$ may be used as an estimator for the inverse Gaussian population, $\hat{\mu}_k$ may not be a valid estimator in the Gaussian case. Furthermore, the moment properties of $\tilde{\mu}_k$ for the inverse Gaussian are not established here.

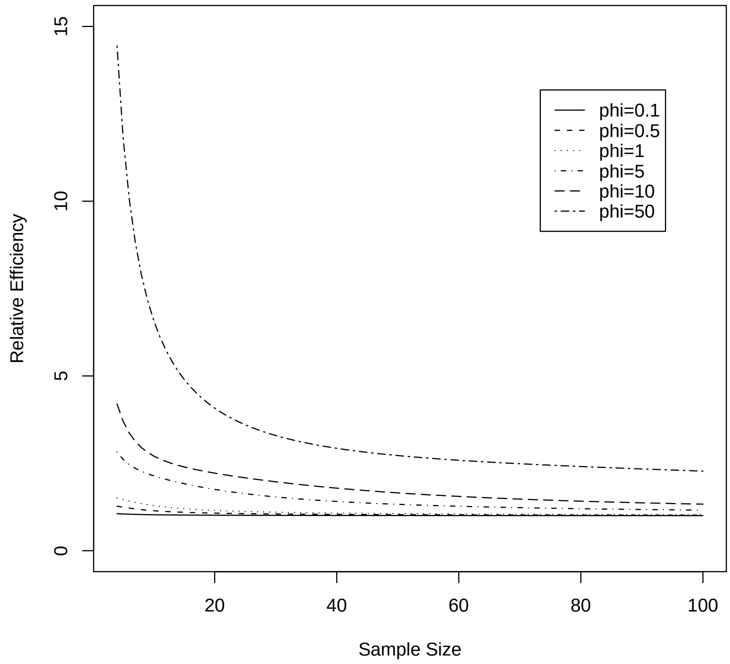

Figure 2: Relative Efficiency of $\hat{\mu}_k, k = 1$

Hence it is not a easy to compare these estimators. Nonetheless we list in Tables 4 and 5 ARB and RE values, respectively, for these estimators for n and ϕ given in Chaubey and Dwivedi (1982), for two cases, (i) $k = 1$ and (ii) $k = (n-1)/n$ for Gaussian and inverse Gaussian populations. Here we used the figures only up to four decimals as to be compatible with those given in Chaubey and Dwivedi (1982).

We note from Table 4 that the ARB values in both the cases are very similar and they resemble more and more as the sample size becomes large. On the other hand, looking at Table 5, we find that the RE values display different characters for Gaussian and inverse Gaussian populations. Where as we find that there may be some loss in efficiency by using the modified estimator in the Gaussian case (see Table 5 for small values of ϕ,) the RE values for the inverse Gaussian case are always greater than 1. There is some what closeness between the respective figures in these tables for very small values of ϕ, which is not very surprising as in this case the inverse Gaussian distribution approximates well to the Gaussian distribution. Another contrasting feature of the Gaussian and inverse Gaussian cases is that the estimator of ϕ for the IG case is unbiased but that for the Gaussian case is not. This may explain the differences in RE values for the two estimators in Table 5.

Table 4: Absolute Relative Bias of $\tilde{\mu}_k$ and $\hat{\mu}_k$ for Gaussian and Inverse Gaussian Populations

n	5		10		30		50		100	
ϕ			$k = \frac{n-1}{n}$							
	(i)	(ii)	(i)	(ii)	(i)	(ii)	(i)	(ii)	(i)	(ii)
0.1	.0385	.0271	.0128	.0113	.0036	.0035	.0021	.0020	.0010	.0010
0.5	.1659	.1193	.0622	.0550	.0179	.0172	.0104	.0102	.0051	.0050
1.0	.2828	.1949	.1197	.1034	.0355	.0343	.0208	.0204	.0102	.0101
5.0	.6534	.3447	.4250	.2635	.1673	.1450	.1016	.0962	.0507	.0500
10.0	.7865	.3759	.6035	.3110	.2998	.2148	.1931	.1622	.0999	.0955
50.0	.9468	.4040	.8871	.3587	.7080	.3165	.5825	.2925	.3935	.2465
ϕ			$k = 1.0$							
	(i)	(ii)	(i)	(ii)	(i)	(ii)	(i)	(ii)	(i)	(ii)
0.1	.0198	.0165	.0100	.0091	0.0033	.0032	.0020	.0020	.0010	.0010
0.5	.0940	.0778	.0492	.0447	0.0166	.0161	.0100	.0098	.0050	.0049
1.0	.1748	.1345	.0963	.0855	0.0332	.0321	.0200	.0196	.0100	.0099
5.0	.5221	.2593	.3721	.2291	0.1578	.1375	.0979	.0929	.0497	.0490
10.0	.6855	.2868	.5526	.2733	0.2859	.2050	.1869	.1573	.0981	.0939
50.0	.9146	.3119	.8662	.3181	0.6957	.3042	.5738	.2855	.3890	.2433

(i): ARB for $\hat{\mu}_k$ for the inverse Gaussian population, (ii): ARB for $\tilde{\mu}_k$ for the Gaussian population

Table 5: Relative Efficiency of $\tilde{\mu}_k$ and $\hat{\mu}_k$ for Gaussian and Inverse Gaussian Populations

n		5		10		30		50		100
ϕ					$k = \frac{n-1}{n}$					
	(i)	(ii)	(i)	(ii)	(i)	(ii)	(i)	(ii)	(i)	(ii)
0.1	1.0371	.8986	1.0313	.9622	1.0103	.9898	1.0061	.9943	1.0030	.9981
0.5	1.1492	.7299	1.1552	.8469	1.0521	.9487	1.0308	.9696	1.0152	.9856
1.0	1.2581	.7348	1.3004	.7790	1.1060	.9035	1.0624	.9409	1.0307	.9703
5.0	2.0596	1.2274	1.9638	.9787	1.5433	.7940	1.3323	.8039	1.1611	.8704
10.0	3.0703	1.5837	2.4370	1.2517	1.9466	.8905	1.6560	.8112	1.3342	.8094
50.0	11.1094	2.2255	6.3122	1.9603	3.2022	1.4819	2.6677	1.2703	2.2595	1.0121
ϕ					$k = 1.0$					
	(i)	(ii)	(i)	(ii)	(i)	(ii)	(i)	(ii)	(i)	(ii)
.1	1.0500	.9474	1.0280	.9719	1.0098	.9907	1.0059	.9946	1.0030	.9981
.5	1.2444	.8206	1.1436	.8786	1.0500	.9530	1.0300	.9712	1.0150	.9860
1.0	1.4632	.8079	1.2902	.8148	1.1019	.9108	1.0609	.9438	1.0303	.9710
5.0	2.6124	1.1786	2.1418	.9836	1.5374	.8031	1.3266	.8100	1.1592	.8731
10.0	3.7010	1.4249	2.7097	1.2222	1.9713	.8950	1.6539	.8159	1.3312	.8094
50.0	11.8139	1.8104	6.5932	1.7587	3.2948	1.4596	2.7222	1.2637	2.2764	1.0121

(i): RE for $\hat{\mu}_k$ for the inverse Gaussian, ii): RE for $\tilde{\mu}_k$ for the Gaussian population

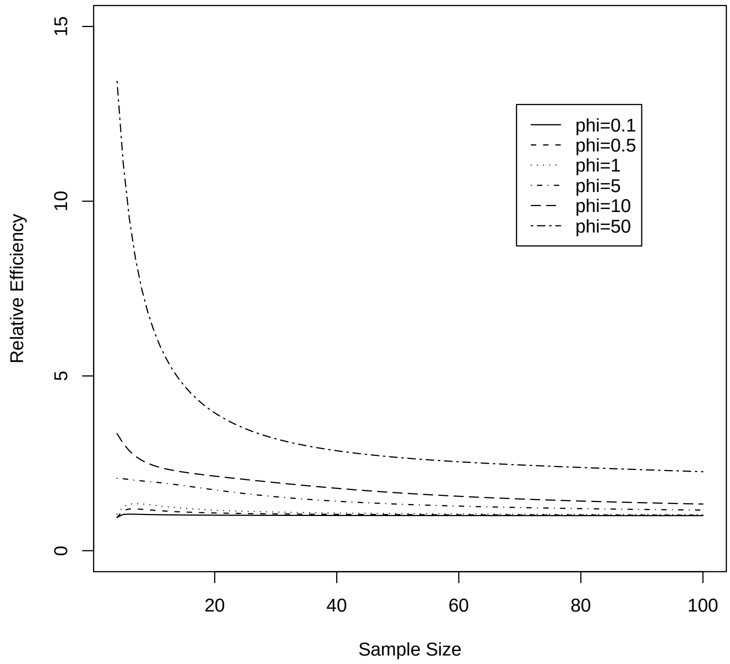

Figure 3: Relative Efficiency of $\hat{\mu}_k, k = \frac{n-1}{n-3}$

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