Journal of Statistical Research *ISSN 0256 - 422 X*
2008, Vol. 42, No. 1, pp. 127-141
Bangladesh

USE OF SUPER–POPULATION MODEL IN SEARCH OF GOOD ROTATION PATTERNS ON SUCCESSIVE OCCASIONS

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summary

Problem of estimating the population mean at current occasion in successive sampling has been studied by various authors in design approach. Present work is an attempt to study the similar kind of problem under a super-population model. Optimum replacement policy and performance of the proposed chain type difference estimator has been discussed under the assumed super-population model. Empirical comparison of the proposed estimator is made with respect to sample mean estimator and suitable recommendations are made.

Keywords and phrases: Super-population, linear model, rotation patterns, auxiliary variate, optimum replacement policy.

AMS Classification: 62D05

1 Introduction

In many surveys, the same population is sampled repeatedly and the same study variate is measured at each occasion, so that the development over time can be followed.

For example in many countries, labor-force surveys are conducted monthly to estimate the number of employed and the rate of unemployment. Other examples are monthly surveys in which data on price of goods are collected to determine a consumer price index, and political opinion survey conducted at regular intervals to measure voter preferences. These practical situations are achieved by means of sampling on successive occasions according to a specified rule, with partial replacement of units. A key issue is the extent to which elements sampled at a previous occasion should be retained in the sample selected at the current occasion; which is termed as optimum replacement policy.

Theory of rotation (successive) sampling appears to have started with the work of Jessen (1942). He pioneered using the entire information collected in previous investigations (occasions). This theory was extended by Patterson (1950), Rao and Graham (1964), Gupta (1979), Das (1982), among others. It is well known fact that an auxiliary variate assists in the estimation of a study variate. Utilizing information on two auxiliary variates, Sen (1971) developed estimators for the population mean on current occasion. Sen (1972, 73) extended his work for several auxiliary variates. Singh et al. (1991) and Singh and Singh (2001) used the auxiliary information on current occasion for estimating the current population mean in two-occasion successive sampling. Singh (2003) extended their work for h-occasions successive sampling. In many situations, information on an auxiliary variate may be readily available on the first as well as on the second occasion, for example tonnage (or seat capacity) of each vehicle or ship is known in survey sampling of transportation, number of polluting industries and vehicles are known in environmental surveys. Many other situations in biological (life) sciences could be explored to show the benefits of the present study. Utilizing the information on an auxiliary variate on both the occasions Feng and Zou (1997), Biradar and Singh (2001), Singh (2005), Singh and Priyanka (2006, 2007 a and 2008) have proposed varieties of chain ratio, difference and regression type estimators for estimating the population mean at current (second) occasion in two-occasion successive sampling.

Works quoted above are design based and consist the use of some known and unknown population parameters. Some times it may be unrealistic to get the ready-made information on unknown population parameters, in such situations it is more realistic to assume a super population model with unknown model parameters, which may be estimated from the available data. Such a model could efficiently link the study and auxiliary variate at different occasions. Motivated with these arguments, Singh and Priyanka (2007b) proposed two different estimators for estimating population mean at current occasion in two-occasion successive sampling under a super-population model.

There are various ways for utilizing the available auxiliary information at estimation stage in successive sampling. Chaining of auxiliary information in two-phase structure is one of them, viz Chand (1975), Kiregyera (1980, 1984) and many others. Successive (rotation) sampling resembles with two-phase sampling. Motivated with these points an estimator of chain-type structure has been proposed under a linear super- population model for estimating population mean at current occasion in two-occasion successive sampling. Assumed super-population model links the study and auxiliary variate over two-occasion. Auxiliary information available at both the occasions is stable over time. It is assumed that under the super-population model, errors are correlated over two occasions and the auxiliary variate is gamma distributed. Results are demonstrated through empirical means of comparison.

2 Notations

Let $U = (U_1, U_2, \ldots, U_N)$ be the N elements finite population, which has been sampled over two-occasion, and the character under study be denoted by y_h $(h = 1, 2)$ on the h^{th} occasion. It is assumed that information on an auxiliary variate z (with known population mean), is available on both the occasions. A simple random sample (without replacement) of n units is taken on the first occasion. A random sub sample of $m = n\lambda$, units is retained (matched) for its use on the second occasion, while a fresh simple random sample (without replacement) of $u = (n - m) = n\mu$, units is drawn on the second occasion from the remaining $(N - n)$ units of the population so that the sample size on the second occasion is n as well. λ and μ $(\lambda + \mu = 1)$ denote the fractions of matched and fresh samples at the second (current) occasion respectively. Under the super-population supposition we assume that the finite population of size N is itself a sample from a super-population. The following notations are considered for the further use:

$\bar{Y}_h$	:	The population mean of the study variate y on the h^{th} $(h = 1, 2)$ occasion.
$\bar{y}_{hn}$	:	The sample mean based on n units on the h^{th} $(h = 1, 2)$ occasion.
$\bar{y}_{hm}$	:	The sample mean on h^{th} $(h = 1, 2)$ occasion, based on m units, which are common to both occasions.
$\bar{y}_{2u}$	:	The sample mean based on u units drawn afresh at second (current) occasion.
$\bar{Z}$	:	The population mean of the auxiliary variate z.
S_y^2, S_z^2	:	The population mean square (variance) of the variates y and z respectively.
$\bar{z}_u$, $\bar{z}_m$	:	The sample means of the auxiliary variate z of the sample sizes shown in suffices.

3 Super-Population Model

It has been assumed that the finite population of size N under consideration is itself a sample from a super-population and the auxiliary variate (z) and study variate (y) are inter-related through a linear model given by

$$y_{hi} = \beta_h z_i + e_{hi}, \quad h = 1, 2 \text{ and } i = 1, 2, \ldots, N \tag{3.1}$$

where β_h $(h = 1, 2)$ are unknown real constants and e_{hi}'s are random errors (disturbances) over h^{th} $(h = 1, 2)$ occasions, such that

$$E_c(e_{hi}|z_i) = 0 \tag{3.2}$$

$$E_c(e_{hi}e_{hj}|z_i, z_j) = 0 \quad \forall\ (i \neq j = 1, 2, \ldots, N) \tag{3.3}$$

$$E_c(e_{hi}^2|z_i) = \delta_h z_i^{g_h}; \quad \delta_h > 0;\ 0 \leq g_h \leq 2 \tag{3.4}$$

$$E_c(e_{hi}e_{h'j}|z_i, z_j) = 0 \quad \forall\ (i \neq j = 1, 2, \ldots, N) \text{ and } h \neq h' = 1, 2 \tag{3.5}$$

$$E_c(e_{hi}e_{h'i}|z_i) = \rho_{e_h e'_h} \sqrt{\delta_h \delta_{h'}}\, z_i^{\frac{g_h + g_{h'}}{2}}; \quad \delta_{h'} > 0;\ 0 \leq g_{h'} \leq 2;\ h \neq h' = 1, 2 \tag{3.6}$$

where $\rho_{e_h e_{h'}}$ is the coefficient of correlation between the random errors (disturbances) over the two-occasion, δ_h and g_h $(h = 1, 2)$ are model parameters.

E_c denotes, the conditional expectation given z_i $(i = 1, 2, \ldots, N)$ and z_i's are independently and identically distributed gamma variates with common density

$$f(z) = \frac{1}{\lceil\theta} e^{-z} z^{\theta - 1}\ ; \quad z \geq 0,\ \theta > 1\,. \tag{3.7}$$

Let us denote the expectation with respect to common distribution of z_i by E_z, model expectation by E_m $(= E_z E_c)$ and design expectation by E_d.

4 Formulation of the Estimator T

To estimate the population mean $\bar{Y}_2$ on the second occasion, two different estimators are suggested. One is difference estimator based on sample of size $u(= n\mu)$ drawn afresh on the second occasion and is given by

$$T_1 = \bar{y}_{2u} + \beta_2(\bar{Z} - \bar{z}_u) \tag{4.1}$$

Second estimator is a chain- type difference to difference estimator based on the sample of size $m(= n\lambda)$ common to both the occasions and is defined as

$$T_2 = \bar{y}_{2m}^* + K(\bar{y}_{1n}^* - \bar{y}_{1m}^*) \tag{4.2}$$

where

$$\begin{aligned} \bar{y}_{2m}^* &= \bar{y}_{2m}^* + \beta_2(\bar{Z} - \bar{z}_m) \\ \bar{y}_{1n}^* &= \bar{y}_{ln} + \beta_1(\bar{Z} - \bar{z}_n) \\ \bar{y}_{1m}^* &= \bar{y}_{1m} + \beta_1(\bar{Z} - \bar{z}_m) \end{aligned}$$

where K is an unknown constant to be determined so as to minimize the variance of the estimator T_2. Combining the estimators T_1 and T_2, the final estimator of $\bar{Y}_2$ is defined as

$$T = \varphi T_1 + (1 - \varphi) T_2 \tag{4.3}$$

where φ is an unknown constant to be determined such that it minimizes the variance of the estimator T.

Remark 1. It is obvious that for estimating the mean on second occasion ignoring findings from earlier occasions, the estimator T_1 is suitable, and it would be appropriate to choose φ to be 1, while for estimating the mean using information on the change from one occasion to the next, the estimator T_2 is emphasized so choosing φ as 0. For asserting both the problems simultaneously, the suitable (optimum) choice of φ is desired.

5 Properties of the Estimator *T*

Theorem 1. *T is an unbiased estimator of $\bar{Y}_2$.*

Proof. Since, T_1 and T_2 are the difference type estimators, so they are unbiased for $\bar{Y}_2$. The final estimator T is a convex linear combination of T_1 and T_2, therefore, T is also an unbiased estimator of $\bar{Y}_2$.

Theorem 2. *Variance of T is obtained as*

$$V(T) = \varphi^2 V(T_1) + (1-\varphi)^2 V(T_2)_{\text{opt}} \,. \tag{5.1}$$

where

$$V(T_1) = \left(\frac{1}{u} - \frac{1}{N}\right)\left(\delta_2 \frac{\overline{\lvert g_2+\theta}}{\overline{\lvert \theta}}\right) \tag{5.2}$$

and

$$V(T_2)_{\text{opt}} = \left(\frac{1}{m} - \frac{1}{N}\right) A + \left(\frac{1}{m} - \frac{1}{n}\right) B \tag{5.3}$$

where

$$A = \delta_2 \frac{\overline{\lvert g_2+\theta}}{\overline{\lvert \theta}}, \quad B = \frac{-\left\{\rho_{e_1e_2}\sqrt{\delta_1\delta_2}\dfrac{\overline{\lvert g+\theta}}{\overline{\lvert \theta}}\right\}^2}{\delta_1 \dfrac{\overline{\lvert g_1+\theta}}{\overline{\lvert \theta}}} \quad \textit{and} \quad g = \frac{(g_1+g_2)}{2} \,.$$

Proof. Since, samples are independent, the variance of T (ignoring the covariance term) is given by

$$V(T) = E(T - \bar{Y}_2)^2 = \varphi^2 V(T_1) + (1-\varphi)^2 V(T_2)$$

$V(T_2)$ is a function of unknown constant K, substituting the optimum value of K (say K_{opt}) the $V(T)$ is give below:

$$V(T) = E(T - \bar{Y}_2)^2 = \varphi^2 V(T_1) + (1-\varphi)^2 V(T_2)_{\text{opt}} \,. \tag{5.4}$$

Under the assumed linear model $y_{hi} = \beta_h z_i + e_{hi}$, $(i = 1, 2, \ldots, N; h = 1, 2)$ we can write

$$\left.\begin{aligned}
\bar{y}_{2u} &= \beta_2 \bar{z}_u + \bar{e}_{2u}, \\
\bar{y}_{1n} &= \beta_1 \bar{z}_n + \bar{e}_{1n}, \\
\bar{y}_{2m} &= \beta_2 \bar{z}_m + \bar{e}_{2m}, \\
\bar{y}_{1m} &= \beta_1 \bar{z}_m + \bar{e}_{1m} \quad \text{and} \\
\bar{Y}_2 &= \beta_2 \bar{Z} + \bar{e}_{2N}
\end{aligned}\right\} \tag{5.5}$$

where

$$\bar{e}_{2u} = (1/u) \textstyle\sum_{i=1}^{u} e_{2i}, \quad \bar{e}_{1n} = (1/n) \sum_{i=1}^{n} e_{1i}, \quad \bar{e}_{2m} = (1/m) \sum_{i=1}^{m} e_{2i},$$
$$\bar{e}_{1m} = (1/m) \textstyle\sum_{i=1}^{m} e_{1i}, \quad \bar{e}_{2N} = (1/N) \sum_{i=1}^{N} e_{2i}.$$

Following Rao (1968), the $V(T_1)$ and $V(T_2)$ have been derived in three steps: First, we operate the design expectation E_d, secondly the conditional expectation E_c and finally the expectation E_z with respect to the distribution of z_i. The design expectation can be evaluated using the results given in Sukhatme et al. (1984).

$$\begin{aligned}
V(T_1) &= E_m E_d [T_1 - \bar{Y}_2]^2 = E_m E_d [\bar{y}_{2u} + \beta_2(\bar{Z} - \bar{z}_u) - \bar{Y}_2]^2 \\
&= E_m E_d [\bar{e}_{2u} - \bar{e}_{2N}]^2 = E_m [V(\bar{e}_{2u})] = E_z E_c \left[\left(\frac{1}{u} - \frac{1}{N} \right) S^2_{e_2} \right] \\
&= E_z E_c \left[\frac{1}{N-1} \left(\frac{1}{u} - \frac{1}{N} \right) \left(\sum_{i=1}^{N} e^2_{2i} - N \bar{e}^2_{2N} \right) \right]
\end{aligned}$$

Using the conditions in equations (3.2)-(3.6) E_c is evaluated and is obtained as

$$V(T_1) = \left(\frac{1}{u} - \frac{1}{N} \right) \frac{1}{N-1} E_z \left[\left(\sum_{i=1}^{N} \delta_2 z_i^{g_2} - \frac{1}{N} \sum_{i=1}^{N} \delta_2 z_i^{g_2} \right) \right]$$

Now, evaluating E_z using the probability distribution of z_i, we have the $V(T_1)$ as in equation (5.2). Similarly,

$$V(T_2) = E_m E_d [T_2 - \bar{Y}_2]^2 = E_m E_d \left[\bar{y}^*_{2m} + K(\bar{y}^*_{1n} - \bar{y}^*_{1m}) - \bar{Y}_2 \right]^2.$$

Applying the results of equation (5.5) and taking expectations in three steps, as discussed in the case of $V(T_1)$, we have $V(T_2)$ as follows:

$$V(T_2) = \left(\frac{1}{m} - \frac{1}{N} \right) \delta_2 \frac{\Gamma(g_2 + \theta)}{\Gamma(\theta)} + \left(\frac{1}{m} - \frac{1}{n} \right) \left[K^2 \delta_1 \frac{\Gamma(g_1 + \theta)}{\Gamma(\theta)} - 2K \rho_{e_1 e_2} \sqrt{\delta_1 \delta_2} \frac{\Gamma(g + \theta)}{\Gamma(\theta)} \right]. \tag{5.6}$$

Since, the variance of the estimator T_2 is a function of unknown constant K, so it is minimized with respect to K and hence the optimum (minimum) value of k is obtained as

$$K_{\text{opt}} = \frac{\rho_{e_1 e_2}\sqrt{\delta_1\delta_2}\dfrac{\lceil g+\theta}{\lceil\theta}}{\delta_1\dfrac{\lceil g_1+\theta}{\lceil\theta}}$$

substituting, the value of K_{opt} in equation (5.6), we get the optimum value of the variance of the estimator T_2 as

$$V(T_2)_{\text{opt}} = \left(\frac{1}{m} - \frac{1}{N}\right)A + \left(\frac{1}{m} - \frac{1}{n}\right)B$$

where $A = \delta_2\dfrac{\lceil g_2+\theta}{\lceil\theta}$, $B = \dfrac{-\left\{\rho_{e_1 e_2}\sqrt{\delta_1\delta_2}\dfrac{\lceil g+\theta}{\lceil\theta}\right\}^2}{\delta_1\dfrac{\lceil g_1+\theta}{\lceil\theta}}$ and $g = \dfrac{(g_1+g_2)}{2}$.

Further substituting the values of $V(T_1)$ and $V(T_2)_{\text{opt}}$ in equation (5.4), we get the expression for the variance of T given as in equation (5.1).

Since, variance of the estimator T in equation (5.1) is a function of the unknown constant φ, it is minimized with respect to φ and subsequently the optimum value of φ is obtained as

$$\varphi_{\text{opt}} = \frac{V(T_2)_{\text{opt}}}{V(T_1) + V(T_2)_{\text{opt}}} \tag{5.7}$$

Substituting this optimum value φ_{opt} in equation (5.1) we obtain the minimum variance of T as

$$V(T)_{\text{opt}} = \frac{V(T_1)V(T_2)_{\text{opt}}}{V(T_1) + V(T_2)_{\text{opt}}} \tag{5.8}$$

Further, substituting the values from equations (5.2) and (5.3) in equation (5.8), the simplified value of $V(T)_{\text{opt}}$ is shown below in Theorem 3.

Theorem 3. *The $V(T)_{\text{opt}}$ is derived as*

$$V(T)_{\text{opt}} = \frac{-fA(fA+B)\mu^2 + \{fA^2 + AB - f(1-f)A^2\}\mu + (1-f)A^2}{fN[(2Af+B)\mu^2 - 2Af\mu + A]} \tag{5.9}$$

where $u\left(=\dfrac{u}{n}\right)$ *and* $f = \dfrac{n}{N}$.

Remark 2. To estimate the population mean on each occasion, a good choice for μ is 1 (the case of no matching) while for estimating the change from one occasion to the other, μ should be 0 (the case of complete matching). To design a strategy that would be efficient for both problems simultaneously, the optimum choice of μ is desired.

Since $V(T)_{\text{opt}}$ is the function of μ (fraction of fresh sample at current occasion), which is an important factor in reducing the cost of the survey, it is necessary to minimize the $V(T)_{\text{opt}}$ in equation (5.9) with respect to μ. The minimum value of μ is obtained as

$$\widehat{\mu} = \frac{-P_2 \pm \sqrt{P_2^2 - P_1}P_3}{P_1} = \mu_0 \quad \text{(say)} \tag{5.10}$$

where

$$\begin{aligned} P_1 &= (A_1A_5 - A_2A_4),\ P_2 = (A_1A_6 - A_3A_4),\ P_3 = (A_2A_6 - A_3A_5), \\ A_1 &= -fA(fA+B),\ A_2 = fA^2 + AB - f(1-f)A^2,\ A_3 = (1-f)A^2, \\ A_4 &= 2Af + B,\ A_5 = -2Af \ \text{ and } \ A_6 = A. \end{aligned}$$

The real values of $\widehat{\mu}$ exists if $(P_2^2 - P_1P_3) \geq 0$. For any combinations of the parameters involved, which satisfies the above condition, two real values of $\widehat{\mu}$ are possible, hence to choose a value of $\widehat{\mu}$, it should be remembered that $0 \leq \widehat{\mu} \leq 1$. All other values of $\widehat{\mu}$ are inadmissible. Substituting the admissible value of $\widehat{\mu}$ from equation (5.10) in equation (5.9) we have

$$V(T)_{\text{opt}^*} = \frac{-fA(fA+B)\mu_0^2 + \{fA^2 + AB - f(1-f)A^2\}\mu_0 + (1-f)A^2}{fN[(2Af+B)\mu_0^2 - 2Af\mu_0 + A]} \tag{5.11}$$

where $V(T)_{\text{opt}^*}$ is the optimum value of T with respect to both the parameters φ and μ.

6 Efficiency Comparison

The percent relative efficiencies of the estimator T with respect to the sample mean estimator $\bar{y}_{2n}$ of the population mean $\bar{Y}_2$ on current occasion, which is based exclusively on a sample of size n on the second occasion, using no information gathered on the first occasion (i.e., case of no matching). Its variance under the assumed model is given by

$$V(\bar{y}_{2n}) = \left(\frac{1}{n} - \frac{1}{N}\right)\left(\beta_2^2\theta + \delta_2 \frac{\Gamma(g_2+\theta)}{\Gamma(\theta)}\right) \tag{6.1}$$

Tables 1–4 present the values of percent relative efficiencies, E and optimum values of μ, i.e., μ_0 of the estimator T over the estimator $\bar{y}_{2n}$ under optimal condition and for a few combinations of the parametric values $\beta_2, g_1, g_2, \theta, \rho_{e_1e_2}, \delta_1$ and δ_2 under the assumed super-population model for given N and n, where $E = \dfrac{V(\bar{y}_{2n})}{V(T)_{opt^*}} \times 100$.

Table 1: Optimum values of μ and percent relative efficiencies of the estimator T over the estimator $\bar{y}_{2n}$ for $(\delta_1 = 1.0, \delta_2 = 2.0, \theta = 8.0, \beta_2 = 0.5)$

$\rho_{e_1e_2}$			0.3		0.6		0.9	
g_1	g_2	$f \downarrow$	μ_0	E	μ_0	E	μ_0	E
0.0	0.0	0.1	0.5118	194.18	0.5556	211.76	0.6964	269.48
		0.3	0.5118	169.29	0.5556	186.67	0.6964	246.50
		0.5	0.5118	137.56	0.5556	153.85	0.6964	213.70
	1.0	0.1	0.5114	109.14	0.5534	118.64	0.6833	148.51
		0.3	0.5114	*	0.5534	104.52	0.6833	135.36
		0.5	0.5114	*	0.5534	*	0.6833	116.75
	2.0	0.1	0.5104	*	0.5481	105.82	0.6540	127.70
		0.3	0.5104	*	0.5481	*	0.6540	115.48
		0.5	0.5104	*	0.5481	*	0.6540	*
1.0	0.0	0.1	0.5114	194.03	0.5534	210.91	0.6833	264.01
		0.3	0.5114	169.14	0.5534	185.81	0.6833	240.64
		0.5	0.5114	137.42	0.5534	153.03	0.6833	207.56
	1.0	0.1	0.5118	109.23	0.5556	119.12	0.6964	151.58
		0.3	0.5118	*	0.5556	105.00	0.6964	138.66
		0.5	0.5118	*	0.5556	*	0.6964	120.21
	2.0	0.1	0.5114	*	0.5537	106.97	0.6846	134.13
		0.3	0.5114	*	0.5537	*	0.6846	122.30
		0.5	0.5114	*	0.5537	*	0.6846	105.54
2.0	0.0	0.1	0.5104	193.64	0.5481	208.74	0.6540	251.89
		0.3	0.5104	168.76	0.5481	183.65	0.6540	227.80
		0.5	0.5104	137.06	0.5481	150.99	0.6540	194.33
	1.0	0.1	0.5114	109.15	0.5537	118.69	0.6846	148.83
		0.3	0.5114	*	0.5537	104.57	0.6846	135.70
		0.5	0.5114	*	0.5537	*	0.6846	117.11
	2.0	0.1	0.5118	*	0.5556	107.35	0.6964	136.61
		0.3	0.5118	*	0.5556	*	0.6964	124.96
		0.5	0.5118	*	0.5556	*	0.6964	108.33

Note: "*" indicates no gain

Table 2: Optimum values of μ and percent relative efficiencies of the estimator T over the estimator $\bar{y}_{2n}$ for $(\delta_1 = 1.0, \delta_2 = 2.0, \theta = 8.0, \beta_2 = 1.5)$

$\rho_{e_1 e_2}$			0.3		0.6		0.9	
g_1	g_2	f	μ_0	E	μ_0	E	μ_0	E
0.0	0.0	0.1	0.5118	970.91	0.5556	1058.8	0.6964	1347.4
		0.3	0.5118	846.46	0.5556	933.33	0.6964	1232.5
		0.5	0.5118	687.79	0.5556	769.23	0.6964	1068.5
	1.0	0.1	0.5114	206.16	0.5534	224.09	0.6833	280.51
		0.3	0.5114	179.72	0.5534	197.43	0.6833	255.68
		0.5	0.5114	146.01	0.5534	162.60	0.6833	220.53
	2.0	0.1	0.5104	108.92	0.5481	117.42	0.6540	141.69
		0.3	0.5104	∗	0.5481	103.30	0.6540	128.14
		0.5	0.5104	∗	0.5481	∗	0.6540	109.31
1.0	0.0	0.1	0.5114	970.15	0.5534	1054.6	0.6833	1320.1
		0.3	0.5114	845.72	0.5534	929.07	0.6833	1203.2
		0.5	0.5114	687.10	0.5534	765.18	0.6833	1037.8
	1.0	0.1	0.5118	206.32	0.5556	225.00	0.6964	286.33
		0.3	0.5118	179.87	0.5556	198.33	0.6964	261.91
		0.5	0.5118	146.15	0.5556	163.46	0.6964	227.06
	2.0	0.1	0.5114	109.15	0.5537	118.69	0.6846	148.83
		0.3	0.5114	∗	0.5537	104.57	0.6846	135.70
		0.5	0.5114	∗	0.5537	∗	0.6846	117.11
2.0	0.0	0.1	0.5104	968.17	0.5481	1043.7	0.6540	1259.5
		0.3	0.5104	843.80	0.5481	918.26	0.6540	1139.0
		0.5	0.5104	685.32	0.5481	754.93	0.6540	971.67
	1.0	0.1	0.5114	206.17	0.5537	224.19	0.6846	281.12
		0.3	0.5114	179.733	0.5537	197.52	0.6846	256.33
		0.5	0.5114	146.02	0.5537	162.69	0.6846	221.21
	2.0	0.1	0.5118	109.23	0.5556	119.12	0.6964	151.58
		0.3	0.5118	∗	0.5556	105.00	0.6964	120.21
		0.5	0.5118	∗	0.5536	∗	0.6964	138.66

Note: "∗" indicates no gain

Table 3: Optimum values of μ and percent relative efficiencies of the estimator T over the estimator $\bar{y}_{2n}$ for $(\delta_1 = 1.0, \delta_2 = 3.0, \theta = 8.0, \beta_2 = 1.5)$

$\rho_{e_1e_2}$			0.3		0.6		0.9	
g_1	g_2	f	μ_0	E	μ_0	E	μ_0	E
0.0	0.0	0.1	0.5118	679.63	0.5556	741.18	0.6964	943.19
		0.3	0.5118	592.52	0.5556	653.33	0.6964	862.76
		0.5	0.5118	481.45	0.5556	538.46	0.6964	747.95
	1.0	0.1	0.5114	169.78	0.5534	184.55	0.6833	231.01
		0.3	0.5114	148.00	0.5534	162.59	0.6833	210.56
		0.5	0.5114	120.24	0.5534	133.91	0.6833	181.61
	2.0	0.1	0.5104	104.89	0.5481	113.09	0.6540	136.44
		0.3	0.5104	*	0.5481	*	0.6540	123.39
		0.5	0.5104	*	0.5481	*	0.6540	105.26
1.0	0.0	0.1	0.5114	679.10	0.5534	738.19	0.6833	924.04
		0.3	0.5114	592.01	0.5534	650.35	0.6833	842.23
		0.5	0.5114	480.97	0.5534	535.62	0.6833	726.46
	1.0	0.1	0.5118	169.91	0.5556	185.29	0.6964	235.80
		0.3	0.5118	148.13	0.5556	163.33	0.6964	215.69
		0.5	0.5118	120.36	0.5556	134.61	0.6964	186.99
	2.0	0.1	0.5114	105.11	0.5537	114.29	0.6846	143.32
		0.3	0.5114	*	0.5537	100.70	0.6846	130.68
		0.5	0.5114	*	0.5537	*	0.6846	112.78
2.0	0.0	0.1	0.5104	677.72	0.5481	730.60	0.6540	881.64
		0.3	0.5104	590.66	0.5481	642.78	0.6540	797.30
		0.5	0.5104	479.73	0.5481	528.45	0.6540	680.17
	1.0	0.1	0.5114	169.79	0.5537	184.63	0.6846	231.51
		0.3	0.5114	148.02	0.5537	162.67	0.6846	211.09
		0.5	0.5114	120.26	0.5537	133.98	0.6846	182.17
	2.0	0.1	0.5118	105.18	0.5556	114.70	0.6964	145.97
		0.3	0.5118	*	0.5556	101.11	0.6964	133.52
		0.5	0.5118	*	0.5556	*	0.6964	115.75

Note: "*" indicates no gain

Table 4: Optimum values of μ and percent relative efficiencies of the estimator T over the estimator $\bar{y}_{2n}$ for $(\delta_1 = 10, \delta_2 = 30, \theta = 8.0, \beta_2 = 2.5)$

$\rho_{e_1e_2}$			0.3		0.6		0.9	
g_1	g_2	f	μ_0	E	μ_0	E	μ_0	E
0.0	0.0	0.1	0.5118	1715.3	0.5556	1870.6	0.6964	2380.4
		0.3	0.5118	1495.4	0.5556	1648.9	0.6964	2177.4
		0.5	0.5118	1215.1	0.5556	1359.0	0.6964	1887.7
	1.0	0.1	0.5114	299.13	0.5534	325.15	0.6833	407.02
		0.3	0.5114	260.76	0.5534	286.46	0.6833	370.99
		0.5	0.5114	211.86	0.5534	235.93	0.6833	319.99
	2.0	0.1	0.5104	119.23	0.5481	128.53	0.6540	155.10
		0.3	0.5104	103.91	0.5481	113.08	0.6540	140.27
		0.5	0.5104	*	0.5481	*	0.6540	119.66
1.0	0.0	0.1	0.5114	1713.9	0.5534	1863.0	0.6833	2332.1
		0.3	0.5114	1494.1	0.5534	1641.4	0.6833	2125.6
		0.5	0.5114	1213.9	0.5534	1351.8	0.6833	1833.4
	1.0	0.1	0.5118	299.36	0.5556	326.47	0.6964	415.45
		0.3	0.5118	260.99	0.5556	287.78	0.6964	380.02
		0.5	0.5118	212.07	0.5536	237.18	0.6964	329.45
	2.0	0.1	0.5114	119.48	0.5537	129.92	0.6846	162.91
		0.3	0.5114	104.16	0.5537	114.47	0.6846	148.5
		0.5	0.5114	*	0.5537	*	0.6846	128.19
2.0	0.0	0.1	0.5104	1710.4	0.5481	1843.9	0.6540	2225.1
		0.3	0.5104	1490.7	0.5481	1622.3	0.6540	2012.2
		0.5	0.5104	1210.7	0.5481	1333.7	0.6540	1716.6
	1.0	0.1	0.5114	299.15	0.5537	325.30	0.6846	407.90
		0.3	0.5114	260.79	0.5537	286.61	0.6846	371.92
		0.5	0.5114	211.88	0.5537	236.07	0.6846	320.97
	2.0	0.1	0.5118	119.56	0.5556	130.39	0.6964	165.93
		0.3	0.5118	104.24	0.5556	114.94	0.6964	151.78
		0.5	0.5118	*	0.5556	*	0.6964	131.58

Note: "*" indicates no gain

7 Conclusion

The following conclusions can be made from Tables 1-4:

(i) It is apparent from the values of E in the tables that the estimator T is far better than the sample mean estimator $\bar{y}_{2n}$ for smaller values of g_1 and g_2.

(ii) The percent relative efficiencies E is maximum in all the tables when we deal with constant variance model (i.e., g_1 or $g_2 = 0.0$).

(iii) If other parameters are same, the value of E increases with the increasing values of β_2. However, μ_0 is unaffected with the change in β_2.

(iv) The percent relative efficiency and the optimum value of μ, is unaffected by any choice of β_1.

Thus, it is clear that the use of auxiliary variate through a super-population linear model is highly rewarding in terms of the proposed estimator. The proposed estimator may be recommended for its practical use by survey statisticians.

Acknowledgements

Authors are thankful to the referee for his valuable and inspiring suggestions. Authors are also thankful to the Indian School of Mines University, Dhanbad for providing the financial assistance to carry out the present work.

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