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ITEM FAILURE DATA OF WEIBULL FAILURE MODEL UNDER BAYESIAN ESTIMATION

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Summary

The objective of the paper is to study the properties of Bayes estimates of Reliability function and Hazard rate under the symmetric and asymmetric loss functions when item failure data are available from the Weibull failure model. The Bayes predictive interval is also determined.

Keywords and phrases: Reliability function; Hazard rate; Bayes predictive interval.

AMS Classification: 62A15, 62F15.

1 Introduction

The probability density function of the Weibull failure model is given by

$$f\left(x;\, v,\theta\right) = \frac{v}{\theta}\, x^{v-1}\, \exp\left\{-\frac{x^v}{\theta}\right\}\,;\; x>0,\; v>0,\; \theta>0. \tag{1.1}$$

where the parameters v and θ are referred to as the shape and the scale parameters respectively.

For the special case $v = 1$, the Weibull failure model is the Exponential distribution. For $v = 2$, it is Rayleigh distribution. For the values in the range $3 \le v \le 4$, the shape of the distribution is close to that of Normal distribution and for a large value of v, say $v \ge 10$, is close to that of smallest extreme value distribution.

The application of the Weibull failure model in life - testing problems and survival analysis has been widely advocated by several authors. Whittemore and Altschuler (1976) used it as a model in biomedical applications. It also has been used as model with diverse types of items such as ball bearing (Lieblein and Zelen, 1956), vacuum tube (Kao, 1959), and electrical isolation (Nelson, 1972). Mittnik and Reachev (1993) found that the Weibull distribution might be adequate statistical model for stock returns. Recently, Hisada and Arizino (2002) discussed about the Reliability tests for Weibull distribution with varying shape parameter.

The squared error loss function (SELF) has been considered as equal weightage to the positive and negative errors for the estimation. Varian (1975) proposed an asymmetric loss function known as the LINEX (linear - exponential) loss function. The invariant form of the LINEX loss function (ILLF) for any parameter θ is defined as

$$L\left(\Delta\right) = \left\{e^{a\Delta} - a\Delta - 1\right\} \; ; \quad a \neq 0, \Delta = \left(\frac{\hat{\theta}}{\theta} - 1\right), \tag{1.2}$$

where a is the shape parameter and $\hat{\theta}$ is an estimate of the parameter θ.

The ILLF is convex, and the shape of this loss function is determined by the value of $'a'$ (the negative (positive) value of $'a'$, gives more weight to overestimation (underestimation)) and its magnitude reflects the degree of asymmetry. It is seen that, for $a = 1$, the function is quite asymmetric with overestimation being costly than underestimation. For small values of $|a|$, the ILLF is almost symmetric and is not far from the SELF.

The natural family of conjugate prior of θ (when shape parameter v is known) is taken as the inverted Gamma distribution with probability density function

$$g_1\left(\theta\right) = \frac{\beta^{\alpha}}{\Gamma\alpha}\,\theta^{-\alpha-1}\,\exp\left(\frac{-\beta}{\theta}\right) \; ; \; \alpha, \beta > 0, \; \theta > 0. \tag{1.3}$$

Further, in a situation where the researchers have no or very little prior information about the parameter θ, one may use a family of priors defined as

$$g_2\left(\theta\right) = \theta^{-d}\,\exp\left(-\frac{cd}{\theta}\right) \; ; \; d, c > 0, \; \theta > 0. \tag{1.4}$$

If $d = 0$, we get a diffuse prior and if $d = 1, c = 0$ a non-informative prior is obtained. For a set of values of d and c that satisfies the equality $\Gamma(d-1) = (cd)^{d-1}$ makes $g_2\left(\theta\right)$ as a proper prior.

The present article studies the properties of the Bayes estimates of the Reliability function and the Hazard rate. A number of authors have extensively studied the properties of the estimates under the Bayesian setup. Few of them are Sinha (1985), Siu and Kelly (1998), Singh and Saxena (2005), Prakash and Singh (2008) and others.

The present paper also predicts the nature of the future observation when sufficient information of the past and the present behavior of an event or an observation is available. A good deal of literature is available on predictive inference for future failure distribution. Nigm (1989), Dellaportas and Wright (1991) and others have discussed the prediction problems in the Weibull distribution. Aitchison and Dunsmore (1975), Bain (1978), Howlader (1985), Raqab (1997), Fernandez (2000), Raqab and Madi (2002), Nigm et al. (2003), Mousa et al. (2005) and Ahmed et al. (2007) are few of those who have been extensively studied predictive inference for the future observations.

2 The Posterior Density

Suppose n items are put to test under model (1.1) without replacement and the test terminates as soon as the r^{th} $(r \le n)$ item fails. If $\underline{x} = (x_1, x_2, ..., x_r)$ be the first r components of the observed failure items, then the likelihood function is obtained as

$$L\left(x_1, x_2, ..., x_r | \theta\right) = \frac{v^r}{\theta^r} \prod_{i=1}^{r} x_{(i)}^{v-1} \; \exp\left(-\frac{rT_r}{\theta}\right), \tag{2.1}$$

where $T_r = \frac{1}{r}\left\{\sum_{i=1}^{r} x_{(i)}^{v} + (n-r)\, x_{(r)}^{v}\right\}$ is an UMVU estimator of the parameter θ and $\frac{2rT_r}{\theta} \backsim \chi^2_{2r}$.

The posterior density of θ (when shape parameter v is known) under prior $g_1(\theta)$ is obtain as

$$Z_1(\theta) = \frac{(rT_r + \beta)^{\alpha+r}}{\Gamma(\alpha + r)} \; \exp\left(-\frac{rT_r + \beta}{\theta}\right) \theta^{-(\alpha+r+1)}. \tag{2.2}$$

Which is again an inverted Gamma distribution with the parameters $(\alpha + r)$ and $(rT_r + \beta)$. Similarly, the posterior density of θ corresponding to $g_2(\theta)$ is given as

$$Z_2(\theta) = \frac{(rT_r + cd)^{d+r-1}}{\Gamma(d + r - 1)} \; \exp\left(-\frac{rT_r + cd}{\theta}\right) \theta^{-(r+d)}. \tag{2.3}$$

This posterior distribution has the same form as the posterior (2.2). The only change is in the place of α and β there is $d - 1$ and cd respectively.

3 The Bayes Estimates of the Reliability Function and Hazard Rate

The Reliability function and Hazard rate for a specific mission time $t\,(> 0)$ (with known v) are obtain as

$$\Psi(t) = P[x > t] = exp\left(-\frac{t^v}{\theta}\right) \tag{3.1}$$

and

$$\rho(t) = \frac{f(t;\, v, \theta)}{\Psi(t)} = \frac{v\, t^{v-1}}{\theta}. \tag{3.2}$$

The Bayes estimates of $\Psi(t)$ and $\rho(t)$ under the SELF corresponding to the posterior $Z_1(\theta)$ are obtained as

$$\Psi_1 = E_P\left(\Psi(t)\right) = \left(1 + \frac{t^v}{rT_r + \beta}\right)^{-(\alpha+r)} \tag{3.3}$$

and

$$\rho_1 = E_P\left(\rho(t)\right) = \frac{v\, t^{v-1}(\alpha + r)}{rT_r + \beta}. \tag{3.4}$$

Here suffix P indicates the expectation taken under posterior density.

The Bayes estimates of of $\Psi(t) = \Psi_2$ (say) and $\rho(t) = \rho_2$ (say) under the ILLF do not exist in the closed form. However, one may obtained them numerically by solving the given equality for the posterior $Z_1(\theta)$

$$E_P\left(\frac{1}{\Psi(t)}\,\exp\left(a\frac{\Psi_2}{\Psi(t)}\right)\right) = e^a\,E_P\left(\frac{1}{\Psi(t)}\right)$$
$$\Rightarrow\ J\left(\eta_1 e^{-a\,\eta_1\,\Psi_2}\right) = e^a\,J(\eta_1) \tag{3.5}$$

and

$$E_P\left(\frac{1}{\rho(t)}\,\exp\left(a\frac{\rho_2}{\rho(t)}\right)\right) = e^a\,E_P\left(\frac{1}{\rho(t)}\right)$$
$$\Rightarrow\ J\left(\eta_2 e^{-a\,\eta_2\,\rho_2}\right) = e^a\,J(\eta_2)\,, \tag{3.6}$$

where $J(q') = \int_0^\infty q'\ \exp\left(-\left(rT_r + \beta\right)y\right)\ y^{\alpha+r-1}\,dy$, $\eta_1 = \exp\left(t^v\,y\right)$, $\eta_2 = \frac{1}{v\,t^{v-1}y}$, and q' is a function of y.

The expressions of the risks for these estimators under the SELF and the ILLF risk criteria $R_{(S)}(\Psi_i)\,,\ R_{(L)}(\Psi_i)\,,\ R_{(S)}(\rho_i)$ and $R_{(L)}(\rho_i)\,; i = 1, 2$, do not exist in the closed form. However, the following Example presents the numerical findings here. Further, the suffix S and L indicates respectively the risk considered with respect to the SELF and the ILLF risk criterion.

4 Illustrative Example

Mann and Fertig (1973) give failure times of airplane components subjected to a life test. The Weibull distribution has often been found a suitable model in such situations. The data are item - failure censored: 13 components were place on test and test terminated at time of 10^{th} failure. The failure times (in hours) of the 10 components that failed were

0.22 0.50 0.88 1.00 1.32 1.33 1.54 1.76 2.50 3.00

The expressions of the Bayes estimates of the Reliability function and Hazard rate and their respective risks under both risk criteria involve a, α, β, θ, v, t, T_r and r. For the selected set of values of $a = 0.50(0.50)2.00$; $\alpha = 1.25, 1.50, 2.50, 5.00, 10.00, 20.00$; $\beta = 0.50, 2.00, 5.00, 10.00, 20.00$; $\theta = 04$; $v = 2.00$ with $t = 2.50$ (hours), the numerical findings have been obtained (Tables 01 - 02) and presented here only for $\beta = 10.00$.

Risk of both the Bayes estimates of the reliability function and hazard rate decreases, when β increases under the SELF and the ILLF criterion for the all considered parametric values. Further, when α increases the risk increases for $R_{(S)}(\Psi_1)\,,\ R_{(L)}(\Psi_1)$ and $R_{(S)}(\Psi_2)$ and decreases otherwise except $R_{(L)}(\Psi_2)$ for the all considered parametric values. Similarly, the risk increases for $R_{(L)}(\Psi_1)\,,\ R_{(S)}(\Psi_2)\,,\ R_{(S)}(\rho_1)$ and $R_{(L)}(\rho_2)\,,$ and decreases for $R_{(L)}(\Psi_2)$ and $R_{(S)}(\rho_2)$ when $'a'$ increases.

Remarks

1. All the results discussed in Section (4) hold for the posterior distribution $Z_2(\theta)$ if we substitute $\alpha = (d-1)$ and $\beta = cd$.

2. One may obtain the results for the complete sample case by replacing only the censored sample size r with the complete sample size n.

5 Bayes Prediction Limits

Statistical prediction limits have many applications in quality control and in the reliability problems and the determination of these limits have been extensively investigated. If we want $100\,\varepsilon\%$ prediction limits for an additional observation, say Y, given a random sample $\underline{X} = (x_1, x_2, ..., x_r)$ from the model (1.1), the problem is equivalent to determining the region $R(\underline{X})$ such that $R(\underline{X})$ covers on the average proportion ε of the distribution of Y.

In the context of prediction, we can say that (l, u) is a $100\,(1-\varepsilon)\,\%$ prediction interval for a future observation Y if

$$P\left[l \le Y \le u\right] = 1 - \varepsilon, \tag{5.1}$$

where l and u are lower and upper prediction limits for the random variable Y, and $1-\varepsilon$ is called the confidence prediction coefficient.

Now, the predicative distribution of a future observation Y from the model (1.1) is obtained by simplifying

$$h\left(y|\underline{X}\right) = \int_\theta f\left(y;\ v, \theta\right).Z_1\left(\theta\right)\ d\theta$$

$$\Rightarrow\ h\left(y|\underline{X}\right) = v\, y^{v-1}\ \left(r+\alpha\right) \frac{\left(rT_r+\beta\right)^{r+\alpha}}{\left(rT_r+\beta+y^v\right)^{r+\alpha+1}}. \tag{5.2}$$

A $100\,(1-\varepsilon)\,\%$ equal tail prediction interval is obtained by solving

$$\int_0^l h\left(y|\underline{X}\right) dy = \int_u^\infty h\left(y|\underline{X}\right) dy = \frac{\varepsilon}{2}. \tag{5.3}$$

Hence, the Bayes prediction limits and the Bayes predictive interval are

$$l = \left\{\left(rT_r+\beta\right)\left\{\left(1-\frac{\varepsilon}{2}\right)^{-\frac{1}{\alpha+r}} - 1\right\}\right\}^{\frac{1}{v}}, \tag{5.4}$$

$$u = \left\{\left(rT_r+\beta\right)\left\{\left(\frac{\varepsilon}{2}\right)^{-\frac{1}{\alpha+r}} - 1\right\}\right\}^{\frac{1}{v}} \tag{5.5}$$

and

$$I = l - u. \tag{5.6}$$

6 Numerical Analysis

The expression of the Bayes predictive intervals involves α, β, v, ε, T_r and r. Using the Example considered in Section 4 with similar set of selected values as considered earlier with the values of confidence level $\varepsilon = 99\%, 95\%, 90\%$, and the numerical finding are presented in the Table 03 for $r = 03, 05, 10$.

The Table 03 shows that the Bayes predictive intervals decrease as confidence level ε increases when other parametric values are fixed. Further, the intervals increase (decrease) as $\beta\,(\alpha)$ increases for other fixed parametric values. The increasing trend in the intervals also has been seen when r increases only for $\beta \leq 2.00$ when other parametric values are fixed.

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Table << 01 >> Risks of the Bayes Estimator							
$\Psi(t) = 0.1510$		$\theta = 4$ :: $v = 2$ :: $t = 2.50$ :: $\beta = 10.00$					
a	$\alpha \rightarrow$	1.25	1.50	2.50	5.00	10.00	20.00
	Ψ_1	0.1425	0.1365	0.1148	0.0745	0.0313	0.0055
0.50	$R_{(S)}(\Psi_1)$	14.433	14.484	14.672	15.044	15.499	15.855
	$R_{(L)}(\Psi_1)$	0.0968	0.0972	0.0983	0.1006	0.1034	0.1056
	Ψ_1	0.1425	0.1365	0.1148	0.0745	0.0313	0.0055
1.00	$R_{(S)}(\Psi_1)$	14.433	14.484	14.672	15.044	15.499	15.855
	$R_{(L)}(\Psi_1)$	0.3366	0.3376	0.3414	0.3489	0.3579	0.3650
	Ψ_1	0.1425	0.1365	0.1148	0.0745	0.0313	0.0055
1.50	$R_{(S)}(\Psi_1)$	14.433	14.484	14.672	15.044	15.499	15.855
	$R_{(L)}(\Psi_1)$	0.6651	0.6671	0.6741	0.6880	0.7048	0.7179
	Ψ_1	0.1425	0.1365	0.1148	0.0745	0.0313	0.0055
2.00	$R_{(S)}(\Psi_1)$	14.433	14.484	14.672	15.044	15.499	15.855
	$R_{(L)}(\Psi_1)$	1.0490	1.0519	1.0624	1.0831	1.1081	1.1275
$\rho(t) = 1.3750$							
	ρ_1	1.5469	1.5812	1.7188	2.0625	2.7500	4.1250
0.50	$R_{(S)}(\rho_1)$	7.3051	7.1541	6.5683	5.2322	3.1103	1.0676
	$R_{(L)}(\rho_1)$	0.0510	0.0500	0.0461	0.0371	0.0224	0.0082
	ρ_1	1.5469	1.5812	1.7188	2.0625	2.7500	4.1250
1.00	$R_{(S)}(\rho_1)$	7.3051	7.1541	6.5683	5.2322	3.1103	1.0676
	$R_{(L)}(\rho_1)$	0.1838	0.1803	0.1668	0.1352	0.0830	0.0321
	ρ_1	1.5469	1.5812	1.7188	2.0625	2.7500	4.1250
1.50	$R_{(S)}(\rho_1)$	7.3051	7.1541	6.5683	5.2322	3.1103	1.0676
	$R_{(L)}(\rho_1)$	0.3744	0.3677	0.3411	0.2787	0.1735	0.0718
	ρ_1	1.5469	1.5812	1.7188	2.0625	2.7500	4.1250
2.00	$R_{(S)}(\rho_1)$	7.3051	7.1541	6.5683	5.2322	3.1103	1.0676
	$R_{(L)}(\rho_1)$	0.6063	0.5957	0.5543	0.4560	0.2875	0.1280

Table << 02 >> Risks of the Bayes Estimator							
$\Psi(t) = 0.1510$		$\theta = 4$ :: $v = 2$ :: $t = 2.50$ :: $\beta = 10.00$					
a	$\alpha \rightarrow$	1.25	1.50	2.50	5.00	10.00	20.00
0.50	Ψ_2	0.0655	0.0613	0.0470	0.0242	0.0064	0.0005
	$R_{(S)}(\Psi_2)$	14.442	14.489	14.655	14.924	15.511	15.580
	$R_{(L)}(\Psi_2)$	0.0257	0.0242	0.0186	0.0252	0.0751	1.5379
1.00	Ψ_2	0.0867	0.0812	0.0622	0.0321	0.0085	0.0006
	$R_{(S)}(\Psi_2)$	14.460	14.508	14.680	14.984	15.522	15.703
	$R_{(L)}(\Psi_2)$	0.0242	0.0227	0.0176	0.0139	0.0196	0.2441
1.50	Ψ_2	0.1172	0.1096	0.0841	0.0433	0.0115	0.0008
	$R_{(S)}(\Psi_2)$	14.476	14.524	14.697	15.011	15.531	15.738
	$R_{(L)}(\Psi_2)$	0.0236	0.0221	0.0171	0.0115	0.0107	0.097
2.00	Ψ_2	0.1612	0.1509	0.1157	0.0596	0.0158	0.0011
	$R_{(S)}(\Psi_2)$	14.492	14.539	14.711	15.029	15.552	15.783
	$R_{(L)}(\Psi_2)$	0.0230	0.0216	0.0167	0.0106	0.0076	0.0521
$\rho(t) = 1.3750$							
0.50	ρ_2	1.6502	1.6948	1.8732	2.3192	3.2112	4.9952
	$R_{(S)}(\rho_2)$	15.887	15.884	15.872	15.842	15.781	15.661
	$R_{(L)}(\rho_2)$	0.0480	0.0468	0.0419	0.0311	0.0153	0.0109
1.00	ρ_2	2.1854	2.2445	2.4808	3.0714	4.2527	6.6154
	$R_{(S)}(\rho_2)$	15.851	15.847	15.831	15.791	15.711	15.552
	$R_{(L)}(\rho_2)$	0.1248	0.1199	0.1016	0.0642	0.0318	0.2149
1.50	ρ_2	2.9522	3.0320	3.3511	4.1490	5.7448	8.9363
	$R_{(S)}(\rho_2)$	15.799	15.793	15.772	15.718	15.610	15.396
	$R_{(L)}(\rho_2)$	0.1484	0.1393	0.1081	0.0717	0.2630	3.1989
2.00	ρ_2	4.0630	4.1729	4.6121	5.7102	7.9065	12.298
	$R_{(S)}(\rho_2)$	15.724	15.716	15.687	15.613	15.465	15.172
	$R_{(L)}(\rho_2)$	0.1579	0.1567	0.1520	0.1444	0.1345	0.1286

$Table$ << 03 >> Bayes Prediction Limits								
$v = 2$			α					
β	r	ε	1.25	1.50	2.50	5.00	10.00	20.00
0.50	03	99 %	2.7289	2.5964	2.2022	1.6729	1.2221	0.8757
		95 %	1.9462	1.8638	1.6118	1.2565	0.9368	0.6803
		90 %	1.5975	1.5337	1.3363	1.0522	0.7907	0.5772
	05	99 %	3.0025	2.9163	2.6311	2.1675	1.6856	1.2569
		95 %	2.2200	2.1624	1.9690	1.6455	1.2975	0.9777
		90 %	1.8477	1.8016	1.6464	1.3837	1.0969	0.8299
	10	99 %	4.1667	4.1095	3.9018	3.4936	2.9542	2.3559
		95 %	3.1782	3.1370	2.9870	2.6891	2.2889	1.8372
		90 %	2.6772	2.6435	2.5199	2.2734	1.9401	1.5612
2.00	03	99 %	3.3073	3.1563	2.6770	2.0337	1.4856	1.0646
		95 %	2.3659	2.2656	1.9594	1.5274	1.1388	0.8269
		90 %	1.9419	1.8644	1.6244	1.2789	0.9612	0.7016
	05	99 %	3.3144	3.2096	2.8958	2.3855	1.8552	1.3833
		95 %	2.4434	2.3799	2.1670	1.8110	1.4280	1.0760
		90 %	2.0335	1.9829	1.8119	1.5229	1.2072	0.9134
	10	99 %	4.2680	4.2094	3.9966	3.5785	3.0259	2.4131
		95 %	3.2554	3.2132	3.0596	2.7544	2.3446	1.8818
		90 %	2.7423	2.7076	2.5811	2.3286	1.9872	1.5991
20.00	03	99 %	7.3277	6.9720	5.9132	4.4922	3.2817	2.3516
		95 %	5.2260	5.0046	4.3282	3.3740	2.5154	1.8267
		90 %	4.2897	4.1182	3.5882	2.8252	2.1230	1.5499
	05	99 %	5.8115	5.6448	5.0927	4.1954	3.2626	2.4329
		95 %	4.2971	4.1855	3.8111	3.1851	2.5113	1.8923
		90 %	3.5763	3.4872	3.1866	2.6782	2.1230	1.6065
	10	99 %	5.3349	5.2617	4.9958	4.4731	3.7824	3.0164
		95 %	4.0692	4.0165	3.8245	3.4431	2.9307	2.3523
		90 %	3.4278	3.3845	3.2264	2.9108	2.4841	1.9989