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Balancing of an Inverted Pendulum Using PD Controller

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Abstract

Balancing of an inverted pendulum robot by moving a cart along a horizontal track is a classical problem in the field of Control Theory and Engineering, for the beginners to understand its dynamics. Several physical models can also be simplified as elastic inverted pendulums like rockets and walking robots. Many researchers have been applying different control algorithm and design techniques such as PID Controller, State Space, Neural Network, Genetic Algorithm (GA) even Particle Swam Optimization (PSO), in both digital and analog domain using various sensors. However, this can also be done by using a single potentiometer as a sensor and Proportional Derivative Controller (PD) controller as the design algorithm. The comparison or difference between the reference and the potentiometer generates control signal to drive the system. In this case, it consists of a thin vertical rod attached at the bottom, referred to as pivot point mounted on a mobile toy car. The car, depending upon the direction of the deflection of the pendulum moves horizontally in order to bring the pendulum to absolute rest. The main idea behind this control process is the use of PD (Proportional and Derivative) controller to generate signal to control the speed and direction of the motor. The only sensor used in this project was a potentiometer (pot) which was attached with the pendulum rod. The variation in its resistance causes change in voltage and afterward, it was compared with the reference voltage to generate the appropriate control signal. PROTEUS software was used for circuit simulation, frequency responses of the system were analyzed in MATLAB with different values of K_P and K_D. Finally, to represent the system stability, root locus diagram was drawn using MATLAB.

Key words: Inverted pendulum, PD controller, System stability, Output responses, Root locus

I. Introduction

The one-dimensional swinging inverted pendulum with two degrees of freedom (i.e. the angle of the inverted pendulum and the movement of robot along forward and backward direction) is a popular demonstration of using feedback control to stabilize an open-loop unstable system. The first solution perhaps to this problem was described by Roberge [1] in his aptly named thesis, “The Mechanical Seal". Subsequently, it has been considered by many researchers [2] as a common benchmark for the investigation of automatic control techniques, most of them have been used as a linearization theory in their control schemes. Since the system is inherently nonlinear, it has been using extensively by the control engineers to verify a modern control theory and is also useful in illustrating some of the ideas in nonlinear control domain. In this system, an inverted pendulum is attached to a cart equipped with a motor that drives it along a horizontal track. The thin vertical rod (the pendulum) hinged at the bottom, referred to as pivot point is mounted on a mobile toy car which can move along horizontal direction. The car, depending upon the direction of the deflection of the pendulum (the angle of the inverted pendulum, θ), moves horizontally in order to bring the pendulum to absolute rest in a vertical position. A PD (Proportional and Derivative) controller has been used to generate signal to control the speed and direction of the motor. The only sensor used in this work was a potentiometer (pot), attached to the bottom of the pendulum rod and variation in its resistance causes change in voltage which is then compared with the reference voltage to generate the appropriate control signal. The Mathematical expression was established to find the system transfer function based on Newton’s second law of motion. Simulation of the circuit mechanism was obtained by applying PROTEUS software. MATLAB simulink has been used for closed loop transfer function simulation with different values of K_P (proportional gain) and K_D (Differential gain) to generate output responses of the system. And finally root locus diagram of the system was drawn in MATLAB to show the system stability.

System Dynamics

To find the system’s transfer function a brief description on the modeling of the inverted pendulum is presented in this section. The system consists of an inverted pole hinged on a cart which is free to move in the *x* direction as shown in Fig. 1. In order to obtain the system dynamics, the following assumptions have been made [4-6]:

1. The system starts in an equilibrium state i.e. that the initial conditions are assumed to be zero.
2. The pendulum does not move more than a few degrees away from the vertical to satisfy a linear model.
3. A step input (displacement of the pendulum, θ) is applied to the system.

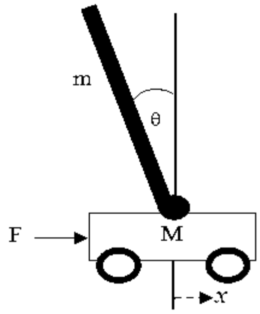

Fig. 1. Cart and Inverted Pendulum System

Table. 1 Parameters of the inverted pendulum

M	mass of the cart	0.3 kg
m	mass of the pendulum	0.2 kg
b	friction of the cart	0.1 N/m/sec
l	length of the pendulum	0.2 m
i	inertia of the pendulum	0.006 kg.m^2
f	force applied to the cart	kg.m/s^2
g	gravity	9.8 m/s^2
θ	Vertical pendulum angle	in degree

For the analysis of system dynamic equations, Newton's second law of motion was applied. Fig. 2 represents the Free Body Diagram (FBD) of the mechanism.

The Force Distribution of the mechanism is shown in Fig. 2. While the pendulum rod tilts with some angle, it resolves two force components along horizontal and vertical direction. 'P' denotes the force exerted by the pendulum in vertical direction, and 'N' in horizontal direction, when θ = 90 □, N= 0, and P = maximum.

From the FBD, summing the forces of the cart along horizontal direction, following equation of motion was obtained [7, 8]:

$$M\ddot{x} + b\dot{x} + N = F \tag{1}$$

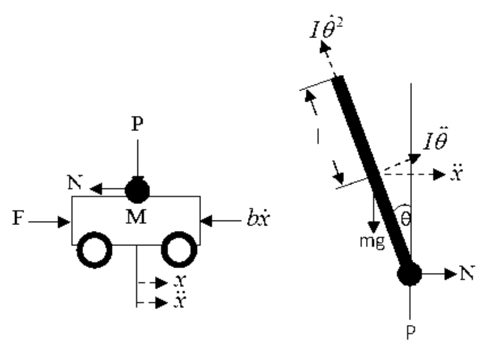

Fig. 2. Free body diagram of the inverted pendulum

Summing the forces along the horizontal direction as shown in the FBD, following equation for N was obtained:

$$N = m\ddot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta \tag{2}$$

After substituting eqn. 2 into eqn. 1, the first equation of motion for the system was found as follows:

$$(M+m)\ddot{x} + b\dot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta = F \tag{3}$$

To acquire the second equation of motion, the forces along the perpendicular direction of the pendulum was summed up and found the following equation:

$$P\sin\theta + N\cos\theta - mg\sin\theta = ml\ddot{\theta} + m\ddot{x}\cos\theta \tag{4}$$

To get rid of P and N terms from the eqn. 4, the moments around the centroid of the pendulum was taken which resulted following equation:

$$-Pl\sin\theta - Nl\cos\theta = l\dot{\theta} \tag{5}$$

Combining eqn. 4 and 5, the second dynamic equation was obtained as follows:

$$(I+ml^2)\ddot{\theta} + mgl\sin\theta = -ml\ddot{x}\cos\theta \tag{6}$$

From eqn. 3 and 6, two linear equations of the transfer function were found, where θ = π. Assume that, θ = π + ø (ø represents a small angle from the vertical upward direction).

Therefore, $\cos\theta = -1, \sin\theta = -\phi$ and $\frac{d^2\theta}{dt^2} = 0$. Thus, after linearization the following 2 equations of motion were appeared (where u represents the input):

$$(I+ml^2)\ddot{\phi} - mgl\phi = ml\ddot{x} \tag{7}$$

$$(M+m)\ddot{x} + b\dot{x} - ml\ddot{\phi} = u \tag{8}$$

To obtain the transfer function of the system analytically, the Laplace transforms of the system equations were taken:

$$(I+ml^2)\Phi(s)s^2 - mgl\Phi(s) = mlX(s)s^2 \tag{9}$$

$$(M+m)X(s)s^2 + bX(s)s - ml\Phi(s)s^2 = U(s) \tag{10}$$

The above two transfer functions can be contracted into a single transfer function as shown below:

$$\frac{\Phi(s)}{U(s)} = \frac{\frac{ml}{q}s}{s^3 + \frac{b(I+ml^2)}{q}s^2 - \frac{(M+m)mgl}{q}s - \frac{bmgl}{q}}$$

Where, $q = [(M+m)(I+ml^2) - (ml)^2]$

$= [(0.3+0.2)\ (0.006+0.2*0.2^2) - (0.2*0.2)^2]$
$= 5.4\times10^{-3}$

$$\frac{\Phi(s)}{U(s)}=\frac{\frac{.2\times.2}{q}s}{s^3+\frac{.1(.006+.2\times.2^2)}{q}s^2-\frac{(.3+.2).2\times9.8\times.2}{q}s-\frac{.1\times.2\times9.8\times.2}{q}}$$

$$\frac{\Phi(s)}{U(s)}=\frac{7.407s}{s^3+0.26s^2-36.296s-7.26}$$

II. Controller Design and Simulation

The Proportional Derivative (PD) is a type of feedback controller whose output, a Control Variable (CV), is generally based on the error (e) between some user-defined Reference Point (RP) and some measured Process Variable (PV). Based on the error each element of the PD controller performs a particular action [3-6].

Proportional (K_p): Hence, error is multiplied by a gain K_p, an adjustable amplifier. In many systems, K_p is responsible for process stability: too low, and the PV will drift away; too high, the PV will oscillate [3].

Derivative (K_d)**:** The rate of change of error multiplied by a gain, K_d. In many systems, K_d is responsible for system response: too high and the PV will oscillate; too low and the PV will respond sluggishly. The designer should also note that derivative action can amplify any noise in the error signal [3]. Tuning of a PD controller involves the adjustment of K_p, and K_d to achieve some user defined "optimal" character of system response.

In this problem, the main concern was to control the pendulum's position, which should return to its original position (vertical) after an initial disturbance, and therefore, the reference signal should be zero. The force applied to the cart was considered as an impulse disturbance. The basic structure of the feedback control system is shown below:

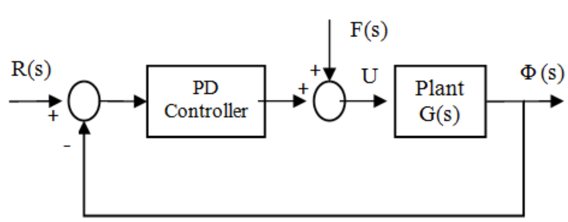

Fig. 3. Control block diagram (close loop) of the inverted pendulum robot

MATLAB simulation: MATLAB Simulink was used for simulation of output responses of the inverted pendulum based on the equation and block diagram aforementioned. In Fig. 4, the block diagram of the simulink is shown below:

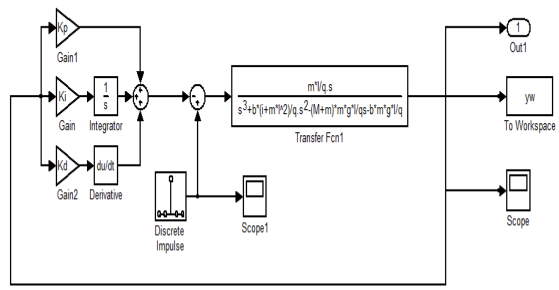

Fig. 4. Block diagram in MATLAB Simulink

The system was examined for different values of K_p and K_d. For instance, three critical combinations of K_p and K_d (Table 2) are presented to understand the entire trend of system stability.

Table. 2. Combinations of K_p and K_d

Case / Gain	Case 1	Case 2	Case 3
K_p	1	1	10
K_d	1	50	50

Case 1: In this case, the values of both K_p and K_d are taken 1. Fig. 5 shows the corresponding MATLAB codes. As can be seen in Fig. 6, the system was unstable as well as the response time was also high (t>8 sec.).

Case 2: In case 2, the value of K_p was increased to 50 keeping the value of K_d unchanged. The output response (Fig. 7) was found stable with some initial oscillations.

Case 3: For case 3, the value of Kp and Kd were considered 10 and 50 respectively. The output response (Fig. 8) was found quite stable and the response time was very low (t<1 sec.)

```
M = 0.3;
m = 0.2;
b = 0.1;
i = 0.006;
g = 9.8;
q = ( M+m)*(i+m*^2) - (m*1)^1;
Kd = 1; Kp = 1; Ki = 0;
[t1,x1,y1] =
sim(‘Inverted_Pendulum_Miniproject’,10);
Plot (t1,y1)
grid
xlabel (‘Kd=1,Kp=1’)
ylabel (‘Output ( Theta)’)
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Fig. 5. MATLAB codes for case 1

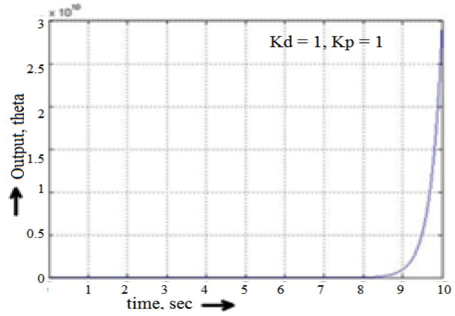


Fig. 6. Response curve for case 1

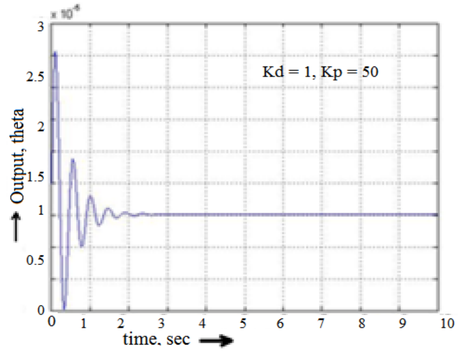


Fig. 7. Response curve for case 2

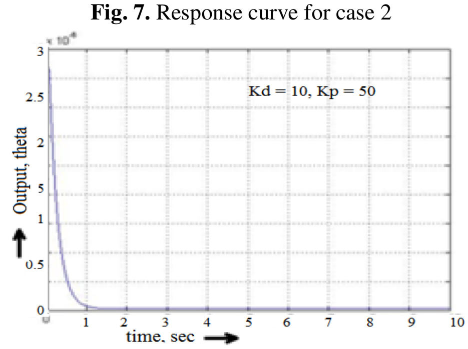


Fig. 8. Response curve for case 3

III. Circuit Analysis and Simulation

Circuit analysis: The rotational direction of the DC servo motor depends on the direction of current which controls the pendulum by moving the cart backward or forward. The current controlling part of the circuit is analyzed in this section.

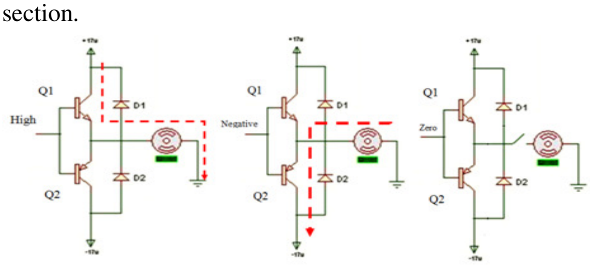


Fig. 9. Direction of current flow of the motor driving circuit

According to Fig. 9, Q_1 and Q_2 are NPN and PNP transistors respectively. When there is a positive voltage at the base of Q_1 and Q_2, the transistor Q_1 goes into forward active region and Q_2 into cutoff. Thus, the current flows from $+V_{CC}$ to ground through motor. Similarly, when there is a negative voltage at the base, Q_2 goes into forward active region and Q_1 into cutoff. Here, current flows from ground to $-V_{CC}$. Table 3 summarizes the functions of Q_1, Q_2, D_1 and D_2 to represent the forward-reverse direction of rotation of the motor.

Table. 3. Function of Q_1, Q_2, D_1 and D_2

Voltage	Component				
	Q_1	Q_2	D_1	D_2	Motor direction
Positive	ON	OFF	OFF	OFF	Forward
Negative	OFF	ON	OFF	OFF	Reverse
Zero	OFF	OFF	OFF	OFF	----------

Darlington transistor was selected in order to drive more current since β value of this transistor is equal to product of β value of two transistors as shown below:

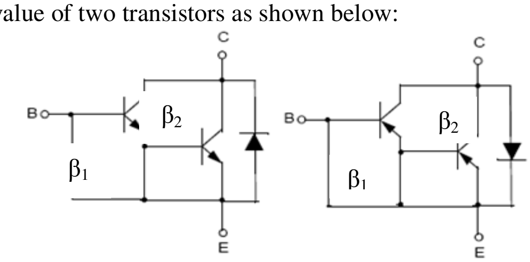


Fig. 10. NPN and PNP Darlington transistor

Here, $\beta_{new} = \beta_1 \times \beta_2$ so that $I_C = I_D \times \beta_{new}$.

Therefore, DC motor rotated faster due to the higher collector current. As the Darlington transistors get heated more, a heat sink was connected.

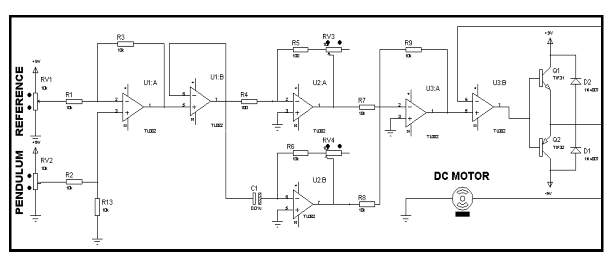


Fig. 11. Snap shot of simulation in PROTEUS

Simulation of circuit: Rather than approaching directly to the circuit fabrication, a simulation was conducted first for successful completion of the project.

PROTEUS software was used for the circuit simulation which helped to visualize the operation of the circuit before building it physically. Different combination of the circuit components such as transistors, operational amplifiers, resistors, DC motor and so on were attempted in PROTEUS to analyze and decide which combination of components results better performance. It enabled to visualize the speed and direction of the DC motor along with appropriate design of the driving circuit. It also facilitated to observe voltage and current at any point of the circuit by using the tools available in PROTEUS (Fig. 11).

According to Fig. 11when the brush of the variable resistor RV2 (pendulum) was moved upwards, the DC motor rotated in clockwise direction. On the other hand, the motor rotated in counter clockwise direction when the brush was moved downwards. The flow of current and rotation of motor were also visible in active simulation. This simulation ensured appropriate functioning of the circuit.

IV.Fabrication

Initially the circuit was assembled on a *'breadboard'* as it is easy to build and test. When it functioned to satisfaction it was soldered onto a *'matrix board'*. Finally a Printed Circuit Board (PCB) was designed and fabricated to assemble the circuitry. The Circuit was driven by an external power supply. The complete structure of the 'Inverted pendulum robot', mounted on a toy car, is shown in Fig. 12.

Rotary type potentiometer was used to sense (position sensor) the angle of the pendulum. The circuit was operated by two separate power supply, viz. ± 12V for the Op-amp circuit and ± 17V for the motor driving circuit.

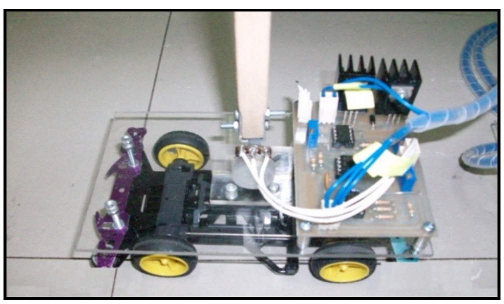
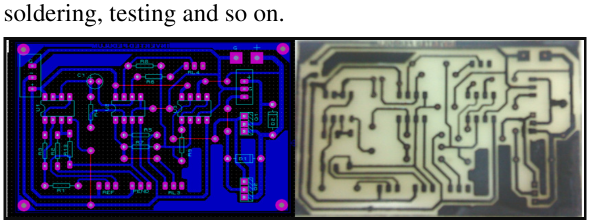


Fig. 12. Inverted pendulum robot

PCB: Printed Circuit Board (PCB), on which the circuit was built, was designed using PROTEUS software and then fabricated as shown in Fig. 13. Unlike matrix board, PCB facilitated to build the circuit more professionally with minimum risk of short-circuits during soldering, de-soldering, testing and so on.



(a)Design view (b) Fabricated view

Fig. 13. Image of PCB

V. Performance Tests Results and Verification

The robot was operated to study its performance. The following results were observed.

- The robot was able to balance the pendulum accurately in both directions. However, the performance was more accurate in forward direction than backward direction due to improper mass balance.
- For small angles of disturbance, the robot was adequately capable to balance the pendulum.
- For larger disturbance, the robot was observed to keep running for reaching balanced position.

Verification: The study was verified through *'root locus'* diagram drawn in MATLAB. Fig. 14 shows the root locus diagram of the inverted pendulum robot. As is observed in the figure, there was a pole on the right hand side of the graph indicating that the system was absolutely unstable for the low value of gain. In case of increasing the magnitude of gain, the pole began to shift towards left direction, referring that the system was going to be stable. With sufficient amount of enhancement of gain, the system at last approached a fully stable state. The other parameters corresponding are also mentioned in the upper right quadrant of the graph.

Limitation

Proper mass balance can be achieved by rearranging the position of circuit board. Since the circuit board is situated at back side, it moves slowly towards backward. If the circuit board is situated at the middle or in two parts, front and back, the performance of the robot will be same towards both backward and forward directions.

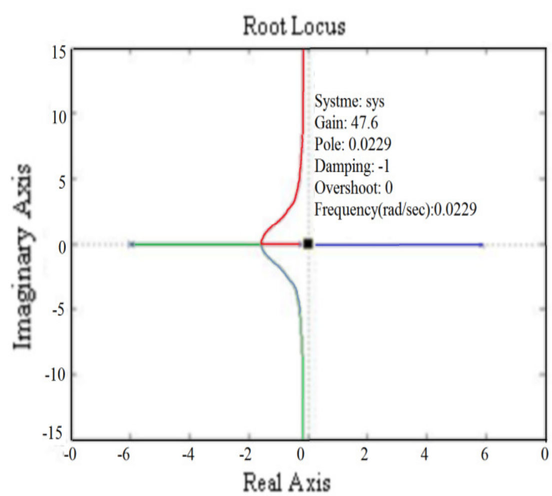


Fig. 14. Root Locus Diagram of the system

VI. Problems Encountered and Solutions

Table. 4. Problems encountered and solutions

Problems	Reasons	Solutions
Inaccurate reference comparison	Friction on potentiometer	Lubricant was used to make pot frictionless
Electrical limitations		
Low speed of motor	Low value of beta in power transistors	Darlington transistor was used
Heating of transistor	As the collector - emitter voltage was high the transistor became heated	Heat sink was used to lower the temperature of the power transistor
Mechanical limitations		
Vibration of the pendulum	Weak joint between pendulum and the pot	Better material was used to strengthen the mechanism
Slip of wheels	Low quality of wheels	Better quality of wheels were tried

VII. Conclusions

The swinging inverted pendulum robot was successfully balanced through the movement of the car along to and fro horizontal direction using a simple PD controller. The frequency responses of the system for different proportional and derivative gains were studied. A Root Locus diagram was drawn to verify the system stability. However, this technique has some limitations from mechanical and electrical points of view such as vibrations, slip of wheels, insufficient current, and heating of transistor. Use of accurate and multiple sensors can increase the accuracy and robustness of the system. Several future enhancements can be made by adopting more sensors like encoder, tilt sensor, accelerometer and so on. In addition, different controllers such as PID, SP, GA may also be used for ensuring better performance. This study may be considered as an interesting initiative for the beginners to have a fundamental perception of Control Theory and Engineering with very limited resources.

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