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A General Construction Method of Simultaneous Confounding in p^n - Factorial Experiment

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Abstract

A general construction method of simultaneous confounding in p^n (p is prime) factorial experiment is proposed. The concept of matrix method in the construction of factorial experiment with a single factorial effect confounded is used to develop the method. The procedure may be extended for the construction of simultaneous confounding of factorial experiment with three or more factorial effects confounded.

I. Introduction

The researchers working with factorial experiments experience difficulty especially when the number of factors as well as the number of levels of each factor is large. It becomes more difficult if we have no required number of homogeneous plots in practice. In such situations, we are bound to use a limited number of homogeneous plots to analyze the factorial effects. As a result, some factorial effects or interactions will be mixed up with block effect, i.e. confounded. Since there is no way to avoid this, the higher order interaction effects are usually considered to be confounded.

Bose and Kishan (1940), Bose (1947) described the construction of p^n factorial designs using finite geometries. The treatments are represented by n-tuples $(a_1,\cdots\cdots,a_n)$ where a_i are elements of $GF(p)$. The method is available only when p is prime or prime power. A system of simultaneous confounding in 2^n factorial experiment has been described, where an intrablock subgroup is constructed with the common elements taken from the factorial effects of two incomplete blocks, each confounded with a single factorial effect (Kempthorne, 1947, 1952). Das (1964) described an equivalent method of Bose in which some of the treatment factors are designated as basic factors and the others as added factors. Levels of added factors are derived by combination of the levels of the basic factors over $GF(p)$. White and Hultquist (1965) extended the field method to design with number of levels of treatment factors. John and Dean (1975) described the construction of a particular class of single replicate block designs, which they call generalized cyclic designs. The essential feature of the method is that the n-tuples giving the treatments of a particular block constitute an Abelian group, the intrablock subgroup. Patterson (1976) described a general computer algorithm, called DSIGN, in which levels of treatment factors are derived by linear combinations of levels of plot and block factors. The method provides finite-field, generalized cyclic and other designs. Mallick, S. A. (1973 & 1975) developed two systems of designing factorial effects with simultaneous confounding of two effects, one for 3^n and the other for 4^n - factorial experiments. In these systems of simultaneous confounding, the combination of levels was based on some manipulating manner. Jalil, et. al. (1990) developed a matrix method of designing a single factorial effect confounded in a P^n - factorial experiment, where the level combinations are obtained by matrix operations of the levels. Construction method of simultaneous confounding has been developed independently for 3^n and 5^n factorial experiments (Jalil and Mallick, 2010). The present work is a general method of construction for simultaneous confounding in a p^n - (p is prime) factorial experiment.

II. Notation and Definition

The formulae given below have been used in determining the number of incomplete blocks or intrablocks (b) and the number of homogeneous plots in each incomplete block or intrablock (k) for a single or simultaneously confounded factorial experiment (symmetrical).

Let b be the number of incomplete blocks or intrablocks in a confounded experiment; and k be the number of plots in each incomplete block or intrablock. Then,

$b=\frac{p^n}{p^{n-r}}$ and $k=\frac{p^n}{p^r}$; where r is the number of factorial effects to be confounded.

For a single factorial effect confounding in p^n factorial experiment, the number of incomplete blocks is computed as, $b=\frac{p^n}{p^{n-1}}=p$ and the number of combinations of levels in each incomplete block is given by, $k=\frac{p^n}{p}=p^{n-1}$.

For a simultaneous confounding (two factorial effects) in p^n factorial experiment, the number of intrablock subgroups is computed as, $b=\frac{p^n}{p^{n-2}}=p^2$ and the number of level combinations in each intrablock subgroup is given by, $k = \frac{p^n}{p^2} = p^{n-2}$.

The general construction matrix with its intrablock subgroups of simultaneous confounding in a p^n factorial experiment can be represented as,

$$\begin{bmatrix} B_1 & B_{p+1} & \cdots & B_{(p-1)p+1} \\ B_2 & B_{p+2} & \cdots & B_{(p-1)p+2} \\ \vdots & \vdots & \vdots & \vdots \\ & & \cdots & \\ B_p & B_{2p} & \cdots & B_{p^2} \end{bmatrix}; \text{ and}$$

in particular, the construction matrices for 3^n and 5^n factorial experiments are given respectively by,

$$\begin{bmatrix} B_1 & B_4 & B_7 \\ B_2 & B_5 & B_8 \\ B_3 & B_6 & B_9 \end{bmatrix} \text{ and } \begin{bmatrix} Col.1 & Col.2 & Col.3 & Col.4 & Col.5 \\ B_1 & B_6 & B_{11} & B_{16} & B_{21} \\ B_2 & B_7 & B_{12} & B_{17} & B_{22} \\ B_3 & B_8 & B_{13} & B_{18} & B_{23} \\ B_4 & B_9 & B_{14} & B_{19} & B_{24} \\ B_5 & B_{10} & B_{15} & B_{20} & B_{25} \end{bmatrix}.$$

III. Construction Method of Simultaneous Confounding in p^n Factorial Experiments

In the construction of p^n (p is prime) factorial experiment with a single factorial effect confounded, we can write the level combinations by the matrix method described below (Jalil, et. al., 1990).

$$M = [M_0, M_1, M_2, \cdots, M_{p-1}]; \quad (1)$$

where incomplete blocks $M_u; u = 0,1,2,\cdots,(p-1)$ is given by:

$M_u = [V_1\{p^0\}, V_2\{p^1\}, \cdots, V_n\{p^{p-1}\}, a_u]_{p^{n-1}\times n}$, with

$V_i\{p^{i-1}\} = p^{i-1}[0 I_{p^{(n-1)-i}}, 0 I_{p^{(n-1)-i}}, \cdots, (p-1) I_{p^{(n-1)-i}}]$, each is a column vector of dimension p^{n-1}.

$\{p^j\} = p^j$ – times repetitions of the elements of $V_i's$ in ascending ordered level;

$u = 0,1,2,\cdots,(p-1);\ i = 1,2,3,\cdots,(n-1);$

$j = 0,1,2,\cdots,(p-1);$ with the restriction that $i = j+1$;

I_m : sum vector of dimension m; and

$a_u = [a_{u1}, a_{u2}, \cdots, a_{up}]'$ is called the adjustment vector.

From the equation (1), M_0 is called the key incomplete block of a single factorial effect confounded in a p^n factorial experiment. For a plan of simultaneous confounding of two factorial effects in a p^n- factorial experiment, we are to perform the following steps.

Step 1. Find the independent key incomplete blocks for the factorial effects to be confounded simultaneously using Equation 1. Let these key incomplete blocks be denoted by, M_0, which represents the level combinations of the key incomplete block for the first confounded factorial effect and M_0' represents the level combinations of the key incomplete block for the second confounded factorial effect.

Step 2. Find the common elements of level combinations (row vectors) of these two key incomplete blocks and form a matrix, can be denoted by B_1. B_1 is called the key intrablock subgroup of level combinations of two factorial effects confounded simultaneously. It can be seen that the key intrablock subgroup contains the combination of the lowest levels for all the factors.

Step 3. To find all other intrablock subgroups of the construction matrix we will follow the computations procedures described below.

The intrablock subgroups $B_2, B_3, \cdots, B_p$, below the key block B_1 are obtained by adding the vectors $(\cdots 0\ c\ 0); c = 1,2,\cdots,p$ with each of the elements (row vectors) of the key intrablock subgroup B_1 as described below.

B_2 is obtained by an addition of the vector $(\cdots 0\ 1\ 0)$ with each of the vector elements of B_1;

B_3 is obtained by an addition of the vector $(\cdots 0\ 2\ 0)$ with each of the vector elements of B_1;

… … …

B_p is obtained by an addition of the vector $(\cdots 0\ p\ 0)$ with each of the vector elements of B_1.

$B_1, B_2, B_3, \cdots, B_p$ are the intrablock subgroups placed in the first column of the construction matrix.

After having the intrablock subgroups in the first column we will get the intrablock subgroups $B_{p+1}, B_{p+2}, \cdots, B_{2p}$ of the second column, which can be computed as shown below.

B_{p+1} is obtained by an addition of the vector $(\cdots 0\ 0\ 1)$ with each of the vector elements of B_1;

B_{p+2} is obtained by an addition of the vector $(\cdots 0\ 0\ 1)$ with each of the vector elements of B_2;

B_{p+3} is obtained by an addition of the vector $(\cdots 0\ 0\ 1)$ with each of the vector elements of B_3;

… … …

B_{2p} is obtained by an addition of the vector $(\cdots 0\ 0\ 1)$ with each of the vector elements of B_p.

The intrablock subgroups $B_{2p+1}, B_{2p+2}, \cdots, B_{3p}$ placed in the third column can be computed as shown below.

B_{2p+1} is obtained by an addition of the vector $(\cdots 0\ 0\ 2)$ with each of the vector elements of B_1;

B_{2p+2} is obtained by an addition of the vector $(\cdots 0\ 0\ 2)$ with each of the vector elements of B_2;

B_{2p+3} is obtained by an addition of the vector $(\cdots 0\ 0\ 2)$ with each of the vector elements of B_3;

… … …

B_{3p} is obtained by an addition of the vector $(\cdots 0\ 0\ 2)$ with each of the vector elements of B_p.

… … …

Proceeding in this way, we will get the intrablock subgroups $B_{(p-1)p+1}, B_{(p-1)p+2}, \cdots, B_{p^2}$ placed in the last column of the construction matrix, which can be obtained as:

$B_{(p-1)p+1}$ is obtained by an addition of the vector $(\cdots 0\ 0\ (p-1))$ with each of the vector elements of $B_{(p-2)p+1}$;

$B_{(p-1)p+2}$ is obtained by an addition of the vector $(\cdots 0\ 0\ (p-1))$ with each of the vector elements of $B_{(p-2)p+1}$;

… … …

B_{p^2} is obtained by an addition of the vector $(\cdots 0\ 0\ (p-1))$ with each of the vector elements of $B_{(p-1)p}$;

Thus, in the construction matrix, we get p^2 intrablock subgroups in p columns. The method is illustrated with two examples described in the section below.

IV. Illustrations

Example 1. Suppose we like to construct the layout of a 3^4 - factorial experiment where the factorial effects $ABCD$ and $ABCD^2$ are confounded simultaneously.

The plan is given by matrix,

$M = [M_0\ \ M_1\ \ M_2]_{27\times 12}$;

$M_u = [V_1\{3^0\}\ V_2\{3^1\}\ V_3\{3^2\}\ a_u]$; $u = 0, 1, 2$; with

$V_1\{1\} = 1[0I_9\ 1I_9\ 2I_9]_{27\times 1}$; $V_2\{3\} = 3[0I_3\ 1I_3\ 2I_3]_{27\times 1}$ and $V_3\{9\} = 9[0I_1\ 1I_1\ 2I_1]_{27\times 1}$.

The adjustment vector, $a_u = [a_{u1}\ a_{u2} \cdots a_{u\,27}]'$ could be obtained by solving the symbolic equation corresponding to the factorial effect to be confounded.

Step 1. Find the matrices M_0 and M_0', which are the key incomplete blocks confounded with $ABCD$ and $ABCD^2$ respectively in a 3^4 - factorial experiment.

$$M_0 = \begin{bmatrix} 0&0&0&0\\ 0&0&1&2\\ 0&0&2&1\\ 0&1&0&2\\ 0&1&1&1\\ 0&1&2&0\\ 0&2&0&1\\ 0&2&1&0\\ 0&2&2&2\\ 1&0&0&2\\ 1&0&1&1\\ 1&0&2&0\\ 1&1&0&1\\ 1&1&1&0\\ 1&1&2&2\\ 1&2&0&0\\ 1&2&1&2\\ 1&2&2&1\\ 2&0&0&1\\ 2&0&1&0\\ 2&0&2&2\\ 2&1&0&0\\ 2&1&1&2\\ 2&1&2&1\\ 2&2&0&2\\ 2&2&1&1\\ 2&2&2&0 \end{bmatrix} \qquad M_0' = \begin{bmatrix} 0&0&0&0\\ 0&0&1&1\\ 0&0&2&2\\ 0&1&0&1\\ 0&1&1&2\\ 0&1&2&0\\ 0&2&0&2\\ 0&2&1&0\\ 0&2&2&1\\ 1&0&0&1\\ 1&0&1&2\\ 1&0&2&0\\ 1&1&0&2\\ 1&1&1&0\\ 1&1&2&1\\ 1&2&0&0\\ 1&2&1&1\\ 1&2&2&2\\ 2&0&0&2\\ 2&0&1&0\\ 2&0&2&1\\ 2&1&0&0\\ 2&1&1&1\\ 2&1&2&2\\ 2&2&0&1\\ 2&2&1&2\\ 2&2&2&0 \end{bmatrix}$$

Step 2. Selecting the common vector elements from M_0 and M_0', we will get the key intrablock subgroup B_1, which is given by

$$B_1 = \begin{bmatrix} 0&0&0&0\\ 0&1&2&0\\ 0&2&1&0\\ 1&0&2&0\\ 1&1&1&0\\ 1&2&0&0\\ 2&0&1&0\\ 2&1&0&0\\ 2&2&2&0 \end{bmatrix}$$

Step 3. Add the vector $(0\,0\,1\,0)$ to each of the vector elements of the key intrablock subgroup, we get,

$$B_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 2 & 2 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 2 & 0 \\ 1 & 2 & 1 & 0 \\ 2 & 0 & 2 & 0 \\ 2 & 1 & 1 & 0 \\ 2 & 2 & 0 & 0 \end{bmatrix}$$

To obtain the third intrablock subgroup B_3, add the vector $(0\ 0\ 2\ 0)$ to each of the vector elements of the key intrablock subgroup. Thus,

$$B_3 = \begin{bmatrix} 0 & 0 & 2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 2 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 2 & 2 & 0 \\ 2 & 0 & 0 & 0 \\ 2 & 1 & 2 & 0 \\ 2 & 2 & 1 & 0 \end{bmatrix}$$

The intrablock subgroups B_4, B_5, B_6 of second column and B_7, B_8, B_9 of the third (last) column can be computed as shown below.

B_4 is obtained by adding the vector $(0\ 0\ 0\ 1)$ to each of the row vectors of B_1;

B_5 is obtained by adding the vector $(0\ 0\ 0\ 1)$ to each of the row vectors of B_2;

B_6 is obtained by adding the vector $(0\ 0\ 0\ 1)$ to each of the row vectors of B_3; and

B_7 is obtained by adding the vector $(0\ 0\ 0\ 2)$ to each of the vectors of B_1;

B_8 is obtained by adding the vector $(0\ 0\ 0\ 2)$ to each of the vectors of B_2;

B_9 is obtained by adding the vector $(0\ 0\ 0\ 2)$ to each of the vectors of B_3;

Thus, we have the complete layout of nine intrablock subgroups as shown below.

$$\left[\begin{array}{ccccccccccccccc}
 & 0 & 0 & 0 & 0 & & 0 & 0 & 0 & 1 & & 0 & 0 & 0 & 2 \\
 & 0 & 1 & 2 & 0 & & 0 & 1 & 2 & 1 & & 0 & 1 & 2 & 2 \\
 & 0 & 2 & 1 & 0 & & 0 & 2 & 1 & 1 & & 0 & 2 & 1 & 2 \\
 & 1 & 0 & 2 & 0 & & 1 & 0 & 2 & 1 & & 0 & 0 & 2 & 2 \\
[B_1] & 1 & 1 & 1 & 0 & [B_4] & 1 & 1 & 1 & 1 & [B_7] & 0 & 1 & 1 & 2 \\
 & 1 & 2 & 0 & 0 & & 1 & 2 & 0 & 1 & & 0 & 2 & 0 & 2 \\
 & 2 & 0 & 1 & 0 & & 2 & 0 & 1 & 1 & & 0 & 0 & 1 & 2 \\
 & 2 & 1 & 0 & 0 & & 2 & 1 & 0 & 1 & & 0 & 1 & 0 & 2 \\
 & 2 & 2 & 2 & 0 & & 2 & 2 & 2 & 1 & & 0 & 2 & 2 & 2 \\
 & & & & & & & & & & & & & & \\
 & 0 & 0 & 1 & 0 & & 0 & 0 & 1 & 1 & & 0 & 0 & 1 & 2 \\
 & 0 & 1 & 0 & 0 & & 0 & 1 & 0 & 1 & & 0 & 1 & 0 & 2 \\
 & 0 & 2 & 2 & 0 & & 0 & 2 & 2 & 1 & & 0 & 2 & 2 & 2 \\
 & 1 & 0 & 0 & 0 & & 1 & 0 & 0 & 1 & & 1 & 0 & 0 & 2 \\
[B_2] & 1 & 1 & 2 & 0 & [B_5] & 1 & 1 & 2 & 1 & [B_8] & 1 & 1 & 2 & 2 \\
 & 1 & 2 & 1 & 0 & & 1 & 2 & 1 & 1 & & 1 & 2 & 1 & 2 \\
 & 2 & 0 & 2 & 0 & & 2 & 0 & 2 & 1 & & 2 & 0 & 2 & 2 \\
 & 2 & 1 & 1 & 0 & & 2 & 1 & 1 & 1 & & 2 & 1 & 1 & 2 \\
 & 2 & 2 & 0 & 0 & & 2 & 2 & 0 & 1 & & 2 & 2 & 0 & 2 \\
 & & & & & & & & & & & & & & \\
 & 0 & 0 & 2 & 0 & & 0 & 0 & 2 & 1 & & 0 & 0 & 2 & 2 \\
 & 0 & 1 & 1 & 0 & & 0 & 1 & 1 & 1 & & 0 & 1 & 1 & 2 \\
 & 0 & 2 & 0 & 0 & & 0 & 2 & 0 & 1 & & 0 & 2 & 0 & 2 \\
 & 1 & 0 & 1 & 0 & & 1 & 0 & 1 & 1 & & 1 & 0 & 1 & 2 \\
[B_3] & 1 & 1 & 0 & 0 & [B_6] & 1 & 1 & 0 & 1 & [B_9] & 1 & 1 & 0 & 2 \\
 & 1 & 2 & 2 & 0 & & 1 & 2 & 2 & 1 & & 1 & 2 & 2 & 2 \\
 & 2 & 0 & 0 & 0 & & 2 & 0 & 0 & 1 & & 2 & 0 & 0 & 2 \\
 & 2 & 1 & 2 & 0 & & 2 & 1 & 2 & 1 & & 2 & 1 & 2 & 2 \\
 & 2 & 2 & 1 & 0 & & 2 & 2 & 1 & 1 & & 2 & 2 & 1 & 2
\end{array}\right]$$

It is easy to verify that,

$(B_1 + B_2 + B_3)\,Vs.\,(B_4 + B_5 + B_6)\ Vs.\,(B_7 + B_8 + B_9)$; confounds the 1st effect, $ABCD$; (column comparison)

$(B_1 + B_4 + B_7)\,Vs.\,(B_2 + B_5 + B_8)\ Vs.\,(B_3 + B_6 + B_9)$; confounds the 2nd effect, $ABC\,D^2$; (row comparison)

$(B_1 + B_5 + B_9)\,Vs.\,(B_2 + B_6 + B_7)\ Vs.\,(B_3 + B_4 + B_8)$; confounds the 1st generalized effect, $ABCD \times ABC\,D^2 = ABC$ (comparing I-totals); and

$(B_1 + B_6 + B_8)\,Vs.(B_2 + B_4 + B_9)\ Vs.\,(B_3 + B_5 + B_7)$; confounds the 2nd generalized effect, $ABC\,D \times (ABC\,D^2)^2 = D$ (comparing J-totals).

Example 2. Suppose we are to construct a 5^3- factorial experiment where the factorial effects ABC and ABC^2 are confounded simultaneously.

Solution. First, we find the two incomplete blocks M_0 and $M_0^{/}$ corresponding confounded effects ABC and ABC^2.

The key incomplete block M_0 confounded with ABC is given by,

$M_0 = [V_1\{5^0\},\, V_2\{5^1\}, a_u]_{5^{n-1} \times 3}$, with

$V_1\{5^0\} = 5^0[0I_5\ 1I_5\ 2I_5\ 3I_5\ 4I_5]'$; $V_2\{5^1\} = 5[0I_1\ 1I_1\ 2I_1\ 3I_1\ 4I_1]'$ and the adjustment vector a_u can be obtained by solving the symbolic equation $x_1 + x_2 + x_3 = 0 \mod 5$, taking first two values of x_1 and x_2 from the vectors V_1 and V_2 respectively for ABC.

Similarly, the key incomplete block $M_0^{/}$ confounded with ABC^2 is given by,

$M_0^{/} = [V_1\{5^0\}, V_2\{5^1\}, a_u]_{5^{n-1}\times 3}$, with

$V_1\{5^0\} = 5^0[0I_5\ 1I_5\ 2I_5\ 3I_5\ 4I_5]'$; $\quad V_2\{5^1\} = 5[0I_1\ 1I_1\ 2I_1\ 3I_1\ 4I_1]'$

and the adjustment vector a_u can be obtained by solving the symbolic equation $x_1 + x_2 + 2x_3 = 0 \bmod 5$, taking first two values of x_1 and x_2 from the vectors V_1 and V_2 respectively for ABC^2.

$$M_0 = \begin{bmatrix} 0&0&0\\0&1&4\\0&2&3\\0&3&2\\0&4&1\\ \\1&0&4\\1&1&3\\1&2&2\\1&3&1\\1&4&0\\ \\2&0&3\\2&1&2\\2&2&1\\2&3&0\\2&4&4\\ \\3&0&2\\3&1&1\\3&2&0\\3&3&4\\3&4&3\\ \\4&0&1\\4&1&0\\4&2&4\\4&3&3\\4&4&2 \end{bmatrix} \qquad M_0' = \begin{bmatrix} 0&0&0\\0&1&2\\0&2&4\\0&3&1\\0&4&3\\ \\1&0&2\\1&1&4\\1&2&1\\1&3&3\\1&4&0\\ \\2&0&4\\2&1&1\\2&2&3\\2&3&0\\2&4&2\\ \\3&0&1\\3&1&3\\3&2&0\\3&3&2\\3&4&4\\ \\4&0&3\\4&1&0\\4&2&2\\4&3&4\\4&4&1 \end{bmatrix}$$

Step 1.

Select the common elements (vectors) of M_0 and M_0' to find the key intrablock subgroup B_1, as shown below.

$$B_1 = \begin{bmatrix} 0&0&0\\1&4&0\\2&3&0\\3&2&0\\4&1&0 \end{bmatrix}$$

Step 2.

We get the second intrablock subgroup B_2 by an addition of the vector $(0\ 1\ 0)$ to each of the vectors of the key intrablock subgroup; B_2 is placed just below (column side) the key block B_1.

Similarly, we get the third, fourth and fifth intrablocks B_3, B_4 and B_5 by addition the vectors $(0\ 2\ 0)$, $(0\ 3\ 0)$ and $(0\ 4\ 0)$ to each of the vectors of the key intrablock subgroups respectively, shown below.

Step 3.

After getting the intrablock subgroups, B_1, B_2, B_3, B_4 *and* B_5 we get the intrablock subgroups B_6, B_7, B_8, B_9 *and* B_{10} by an addition of the vector $(0\ 0\ 1)$ to each of the vector elements (row vectors) of B_1, B_2, B_3, B_4 *and* B_5 respectively.

Similarly, an addition of the vector $(0\ 0\ 2)$ to each of the vector elements of B_1, B_2, B_3, B_4 *and* B_5 will produce the intrablock subgroups $B_{11}, B_{12}, B_{13}, B_{14}$ and B_{15}.

An addition of the vector $(0\ 0\ 3)$ to each of the vector elements of B_1, B_2, B_3, B_4 *and* B_5 will produce the intrablock subgroups $B_{16}, B_{17}, B_{18}, B_{19}$ and B_{20}; and

An addition of the vector $(0\ 0\ 4)$ to each of the vector elements of B_1, B_2, B_3, B_4 *and* B_5 will produce the intrablock subgroups $B_{21}, B_{22}, B_{23}, B_{24}$ and B_{25}, which completes the plan, shown below.

$$\left[\begin{array}{cccccccccccccccccccc}
 & 0&0&0 & & 0&0&1 & & 0&0&2 & & 0&0&3 & & 0&0&4\\
 & 1&4&0 & & 1&4&1 & & 1&4&2 & & 1&4&3 & & 1&4&4\\
B_1 & 2&3&0 & B_6 & 2&3&1 & B_{11} & 2&3&2 & B_{16} & 2&3&3 & B_{21} & 2&3&4\\
 & 3&2&0 & & 3&2&1 & & 3&2&2 & & 3&2&3 & & 3&2&4\\
 & 4&1&0 & & 4&1&1 & & 4&1&2 & & 4&1&3 & & 4&1&4\\
 \\
 & 0&1&0 & & 0&1&1 & & 0&1&2 & & 0&1&3 & & 0&1&4\\
 & 1&0&0 & & 1&0&1 & & 1&0&2 & & 1&0&3 & & 1&0&4\\
B_2 & 2&4&0 & B_7 & 2&4&1 & B_{12} & 2&4&2 & B_{17} & 2&4&3 & B_{22} & 2&4&4\\
 & 3&3&0 & & 3&3&1 & & 3&3&2 & & 3&3&3 & & 3&3&4\\
 & 4&2&0 & & 4&2&1 & & 4&2&2 & & 4&2&3 & & 4&2&4\\
 \\
 & 0&2&0 & & 0&2&1 & & 0&2&2 & & 0&2&3 & & 0&2&4\\
 & 1&1&0 & & 1&1&1 & & 1&1&2 & & 1&1&3 & & 1&1&4\\
B_3 & 2&0&0 & B_8 & 2&0&1 & B_{13} & 2&0&2 & B_{18} & 2&0&3 & B_{23} & 2&0&4\\
 & 3&4&0 & & 3&4&1 & & 3&4&2 & & 3&4&3 & & 3&4&4\\
 & 4&3&0 & & 4&3&1 & & 4&3&2 & & 4&3&3 & & 4&3&4\\
 \\
 & 0&3&0 & & 0&3&1 & & 0&3&2 & & 0&3&3 & & 0&3&4\\
 & 1&2&0 & & 1&2&1 & & 1&2&2 & & 1&2&3 & & 1&2&4\\
B_4 & 2&1&0 & B_9 & 2&1&1 & B_{14} & 2&1&2 & B_{19} & 2&1&3 & B_{24} & 2&1&4\\
 & 3&0&0 & & 3&0&1 & & 3&0&2 & & 3&0&3 & & 3&0&4\\
 & 4&4&0 & & 4&4&1 & & 4&4&2 & & 4&4&3 & & 4&4&4\\
 \\
 & 0&4&0 & & 0&4&1 & & 0&4&2 & & 0&4&3 & & 0&4&4\\
 & 1&3&0 & & 1&3&1 & & 1&3&2 & & 1&3&3 & & 1&3&4\\
B_5 & 2&2&0 & B_{10} & 2&2&1 & B_{15} & 2&2&2 & B_{20} & 2&2&3 & B_{25} & 2&2&4\\
 & 3&1&0 & & 3&1&1 & & 3&1&2 & & 3&1&3 & & 3&1&4\\
 & 4&0&0 & & 4&0&1 & & 4&0&2 & & 4&0&3 & & 4&0&4
\end{array}\right]$$

V. Conclusion

In this article, a general construction method has been developed for simultaneous confounding in p^n (p is prime) factorial experiment. The construction of simultaneous confounding in p^n factorial experiment becomes easier and rewarding. The method is restricted to p^n factorial experiment when p is prime and it can be extended for a simultaneous confounding of three or more factorial effects.

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