

Determination of Optimal Smoothing Constants for Holt - Winter's Multiplicative Method

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(Received: 11 February 2019; Accepted: 23 June 2019)

Abstract

There are many trade time series parade having seasonality. This paper ponders on taking the appropriate smoothing constants of seasonal time series data using Holt-Winter's exponential smoothing method in demand forecasting of toy production in a company. It is a quantitative technique in forecasting. This forecasting method is used three constants that assign weights to current demands and previous forecasts to decide on a new forecast. We have demonstrated the techniques how to choose these constants by presenting a real life example and calculated corresponding forecast value of this technique for the optimal smoothing constants.

Keywords: Exponential Smoothing; Holt-Winter's Multiplicative Method; Smoothing Constants; Forecast Error.

I. Introduction

Forecasting is a progression of prediction of future based on available past data. It is the first stage of arrangement procedure for any production in the business organization. Many organizations did not have success because of lack of knowledge of appropriate forecasting method. There are various methods to forecast the future demand.

Because of simplicity trend analysis model is very easy to use. However, it is less valid for long-term time series. By using decomposition model we are able to observe the data set in components named as trend, seasonality, cyclic and random. If we have more historical data decomposing trend and seasonality will be easier for us. The Holt – Winter's forecasting method is known as triple exponential smoothing method. This method is ubiquitous, modest to utilize and gives good result in applied applications. So, Holt's – Winter's forecasting method is very important forecasting technique. It has three constants which is assigned weights to current demands and previous forecasts to arrive at new forecasts. Substantial exertion has engrossed on finding the right values to use.

The smoothing constant is very important because it determines how the antique time series values are weighted. Forecasting values are mottled with the values of the smoothing constant. For appropriate demand forecasting choosing right smoothing constant is very significant but there is no consistent guideline to select these constant. In order to estimate the optimal smoothing constants for Holt-Winter's method different values are tried out on past time series, and the ones that minimize some error function like Mean Absolute Deviation (MAD), Mean Squared Error (MSE) and Mean Absolute Percentage Error (MAPE) used for forecast accuracy.

Many authors used different procedure to determine optimal smoothing constants. M Babul Hasan and Md. Nayan Dhali developed a process to determine the optimal

smoothing constant for Exponential smoothing and Holt's method¹. C L Karmaker developed a framework for the selection of optimal smoothing constant of single Exponential smoothing method². Liljana Ferbar Tratar tried to improve Holt's-Winter's method over a case of overnight stays of tourists in republic of Slovenia³. Seng Hansun estimated new rules for unknown parameter on Holt's Winter's multiplicative method⁴. Sanjoy Kumar Paul tried to determine the Exponential smoothing constant by minimizing the Mean Absolute Deviation (MAD) and Mean Square Error (MSE)⁶.

Demand forecasting is very important for a business organization. It helps the company prevent losses by making the proper decisions based on relevant information. Forecasting prevents the organization of failing from spending time and money for developing, manufacturing and marketing. The accuracy of forecast value depends on some constants namely smoothing constant. That's why we try to determine the suitable smoothing constants that minimize the forecasting error for a particular set of data values.

The rest of the paper is organized as follows. In Section II, we describe Holt's Winter's multiplicative forecasting technique. In section III, we discuss how to find out optimal smoothing constants by presenting a real life example. Finally, we draw a conclusion.

II. Holt's Winter's Multiplicative Forecasting Method

The Holt-Winter's multiplicative method of exponential smoothing involves trend and seasonality and is based on three smoothing equations: for level, for trend and for seasonality. The decision regarding depends on time series characteristics as follows:

$$L_t = \alpha \frac{A_t}{S_{t-n}} + (1 - \alpha)(L_{t-1} + T_{t-1})$$

$$T_t = \beta(L_t - L_{t-1}) + (1 - \beta)T_{t-1}$$

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$$S_t = \gamma \frac{A_t}{L_t} + (1 - \gamma)S_{t-n}$$

At the end of period t the forecast $F_{t,k}$ for month $t + k$ is

$$F_{t,k} = (L_t + k T_t)S_{t+k-n}$$

Where, n is the number of seasons, e.g., $n = 4$ for quarterly data and $n = 12$ for monthly data.

- A_t = Actual demand at period t
- L_t = Estimated base label at period t
- T_t = Estimated trend at period t
- S_t = Seasonal index at period t

α, β and γ are smoothing constants and $0 < \alpha < 1, 0 < \beta < 1$ and $0 < \gamma < 1$

III. Data Analysis and Result Discussion

A renowned toys company in Dhaka produced different types of toys. Actual demand (thousands) of these products quarterly in different year is given below. We have to forecast the demand for year 2019 of these products.

Table 1. Demand for different years of toy products

Year	Quarter	Demand
2016	1	144
	2	185
	3	130
	4	94
2017	1	140
	2	190
	3	136
	4	90
2018	1	145
	2	188
	3	130
	4	95

Explanation

To forecast the actual demand we need to select the suitable forecasting method. So first of all we analyze the given data values

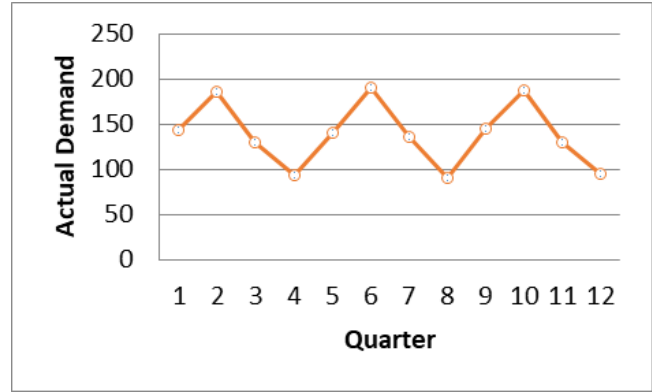


Fig. 1. Variation of actual demand with respect to quarter

We know that if the data has trend may be positive or negative demand forecasting will be calculated by Holt’s method. But if the data has seasonality demand forecasting will be calculated by Holt-Winter’s Multiplicative method.

From the figure-1 we see that a seasonal variation with respect to time be occurred for the actual demand. So Holt’s- Winter’s Multiplicative forecasting method be used to forecast the future demand

The following procedure is given below:

Average Demand of the year 2016 = 138.25

Average Demand of the year 2017 = 139

The estimate of trend $T_0 = \frac{139-138.25}{4} = 0.1875$

Trend value = $\frac{1+2+3+4}{4} = 2.5$

From trend line equation,

Average Demand of the year 2016 = $L_0 + 2.5 * T_0$

The estimate of base level $L_0 = 137.7813$

Initial seasonal index

$S_{-3} = 1.029404$

$S_{-2} = 1.350911$

$S_{-1} = 0.950214$

$S_0 = 0.669471$

Since, In Holt’s-Winter’s Multiplicative method, there has been used three smoothing constants α, β and γ . So, first of all we fixed a particular value of α and β then calculate MAD, MSE & MAPE for the different values of γ . Then keeping the value of α be fixed and changing the values of β , we compute MAD, MSE & MAPE. After that we fixed another value of α then for different values of β and γ , we compute MAD, MSE & MAPE. Continuing this process by fixing a particular value of α and changing the value of β and γ , we get MAD, MSE & MAPE and find out whether MAD, MSE & MAPE give minimum value. The above process is shown by the Table 2 as follows:

Table 2. MAD, MSE & MAPE for Holt's-Winter's Multiplicative method, for changing value of α , β and γ

constant (α)	constant (β)	constant (γ)	MAD	MSE	MAPE
0.1	0.1	0.1	2.295894	6.747395	1.785242
		0.2	2.413528	7.438438	1.876714
		0.4	2.679769	9.1203	2.082165
		0.6	2.987313	11.30845	2.317614
		0.9	3.526133	15.89677	2.727029
	0.3	0.1	2.318772	6.870918	1.802642
		0.3	2.569655	8.394748	1.996776
		0.6	3.026129	11.58918	2.345942
		0.8	3.383921	14.55698	2.617355
	0.6	0.1	2.353655	7.067009	1.829285
		0.2	2.477853	7.814151	1.925271
		0.4	2.759986	9.64328	2.141542
		0.7	3.267581	13.51354	2.526679
	0.8	0.1	2.377916	7.208615	1.847559
		0.3	2.64388	8.864453	2.051894
		0.5	2.956377	11.03796	2.289678
0.9		3.721189	17.69069	2.865572	
0.3	0.1	0.1	2.542493	8.151959	1.982996
		0.2	2.650395	8.842943	2.066979
		0.5	3.022077	11.4196	2.353586
		0.7	3.309846	13.67874	2.573501
	0.3	0.1	2.612425	8.624305	2.038457
		0.2	2.726065	9.375236	2.126854
		0.4	2.979193	11.14579	2.322354
		0.7	3.423491	14.68004	2.662274
	0.5	0.1	2.683976	9.141323	2.09576
		0.3	2.931589	10.86883	2.288311
		0.6	3.372191	14.32474	2.627351
		0.8	3.712037	17.41803	2.886751
0.5	0.1	0.1	2.797651	9.962027	2.186665
		0.4	3.071554	11.99899	2.399532
		0.7	3.386026	14.57949	2.641354
	0.2	0.1	2.851058	10.40948	2.231615
		0.3	3.034589	11.79605	2.374865
		0.7	3.457782	15.32296	2.701483
	0.6	0.1	3.035909	12.35641	2.400954
		0.2	3.133236	13.1965	2.478437
		0.5	3.456038	16.14539	2.733022
0.8	0.1	0.1	3.275989	14.31547	2.535228
		0.2	3.317267	14.70218	2.566484
		0.7	3.532246	16.81817	2.728053
	0.3	0.1	3.481694	16.75154	2.699614
		0.5	3.66493	18.73876	2.836852
		0.9	3.85612	20.98134	2.978424
	0.7	0.1	3.978897	23.94546	3.079212
		0.3	4.089638	25.47878	3.159123
		0.6	4.256246	27.95293	3.277832

Table 2 shows the different value of MAD, MSE & MAPE for the corresponding different values of smoothing constants α, β and γ

To find the optimal value of smoothing constant, minimum values of MAD, MSE & MAPE are selected and corresponding value of smoothing constant is the optimal value for this problem. After analyzing Table 2, we have the following conclusion table

Table 3. Minimum values of MAD, MSE & MAPE and corresponding value of smoothing constants

Criteria	Minimum value	value of α	value of β	value of γ
Mean Absolute Deviation	2.295894	0.1	0.1	0.1
Mean Squared Error	6.747395	0.1	0.1	0.1
Mean Absolute Percentage Error	1.785242	0.1	0.1	0.1

From Table-3 we see that MAD, MSE & MAPE are all provide the minimum value for the same value of smoothing constants $\alpha = 0.1, \beta = 0.1$ and $\gamma = 0.1$; Thus $\alpha = 0.1, \beta = 0.1$ and $\gamma = 0.1$ is our required optimum value of smoothing constants.

Using Holt’s-Winter’s Multiplicative forecasting method the forecast value for the optimal smoothing constants $\alpha = 0.1, \beta = 0.1$ and $\gamma = 0.1$ are given below

Table 4. Actual value and forecast demand for optimal smoothing constants in 2019

Year	Quarter	Demand	Forecast Demand
2016	1	144	142.0256
	2	185	186.9218
	3	130	131.5262
	4	94	92.67705
2017	1	140	143.0876
	2	190	187.1892
	3	136	132.0261
	4	90	93.66953
2018	1	145	143.3607
	2	188	188.8189
	3	130	133.0938
	4	95	93.28859
2019	1	-	144.2625
	2	-	189.5022
	3	-	133.2296
	4	-	94.0878

Table 4 represents the actual value and forecast demand for the optimal smoothing constants and also forecast the demand at the year of 2019.

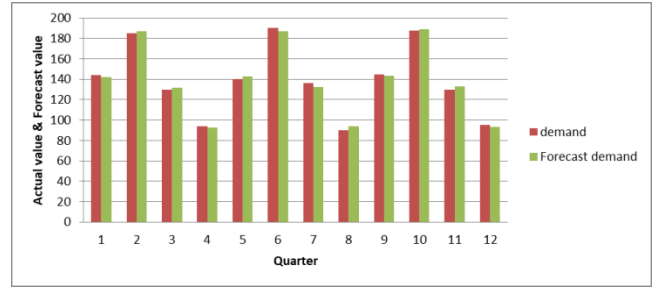


Fig. 2. Comparison of actual demand and forecast value

Figure 2 represents the comparison between actual demand and forecast value for optimal smoothing constant. From Figure 2, we see that, forecast value for every quarter is very close to actual demand

We have explained the procedure of choosing smoothing constants by presenting a real life example. Since our aim is to find out optimal smoothing constants for a particular set of data values, we are successfully able to find out this constant for the given real life problem. Using above procedure, we can compute the optimal value of smoothing constants for any types of data values.

IV. Conclusion

Holt’s-Winter’s Multiplicative method is one of the important forecasting method for seasonal time series data. This technique contains three constants that are needed to smooth the forecast accuracy. In this paper, we developed a procedure how to choose these smoothing constants. We illustrated the choosing procedure by presenting a real life example. We therefore, hope that our procedure can help an organization to compute the optimal value of smoothing constants for particular data values that enhance the accuracy of forecasting.

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