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A General Method of Constructing Layout with Single Factorial Effect Confounded in p^n Factorial Experiments

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Abstract

A general method of constructing layout with single factorial effect confounded in p^n factorial experiments is proposed. It becomes easier to construct the design of confounding a single factor in a p^n factorial experiment, especially when the number of factors as well as the number of levels becomes larger.

Keywords: Adjustment Factor, Adjustment Vector, Confounding.

I. Introduction

When the number of factors as well as the number of levels of each factor is large, it becomes difficult to make homogeneous plots in practice. In such situations, we are bound to use a limited number of homogeneous plots to analyze the factorial effects. As a result, some factorial effects or interactions will be mixed up with block effect, i.e. confounding.

Bose and Kishan[1], Bose[2] described the construction of p^n factorial design using finite geometries. The treatments are represented by n-tuples $(a_1, a_2, \ldots, a_n)$ where a_i are elements of $GF(p)$. The method is available only when p is prime or prime power. Fisher[3] discussed a method to develop the connection of the subject with that of Abelian groups, to prove a general proposition connecting the minimal size of block required with the number of factors involved, and to supply a catalogue of systems of confounding available up to fifteen factors. A system of simultaneous confounding in 2^n factorial experiment has been described, where an intrablock subgroup is constructed with the common elements taken from the factorial effects of two incomplete blocks, each confounded with a single factorial effect[4, 5]. Das[6] described an equivalent method of Bose[2] in which some of the treatment factors are designated as basic factors and the others as added factors. Levels of added factors are derived by combination of levels of the basic factors over $GF(p)$. S. C. Cotter[7] proposed a general method of confounding for symmetrical factorial experiments. This paper considered the problem of conducting a p^n factorial experiment in blocks size p^1. These designs can be constructed for all values of p, although for certain values better confounding patterns are available. Also analysis of these designs was given, showing which components of the sum of squares were confounded with blocks. John and Dean[8] described the construction of a particular class of single replicate block designs, which they call generalized cyclic designs. The essential feature of the method is that the $n-$tuples giving the treatments of a particular block constitute an Abelian group, the intrablock subgroup. Patterson[9] described a general computer algorithm, called DSIGN, in which levels of treatment factors are derived by linear combinations of levels of plot and block factors. The method provides finite-field, generalized cyclic and other designs. Mallick, S. A.[10, 11] developed two systems of designing factorial effects with simultaneous confounding of two effects, one for 3^n and other for 4^n - factorial experiments. In these systems of simultaneous confounding, the combination of levels was based on some manipulating manner. Jalil, *et. al.*[12] developed a matrix method of designing a single factorial effect confounded in a p^n - factorial experiment, where the level combinations are obtained by matrix operations of the levels. This method is applicable only when p is prime. In this article, we make moderation in taking adjustment factor using which we can construct a plan for a factorial effect to be confounded in a p^n factorial experiment, where p is prime or non prime.

II. Method

Consider a matrix M of order $p^{n-1} \times np$, which represents the construction method of a p^n f.e. confounded with a factorial effects as $M = \left[M_0\ M_1 \ldots \ldots M_{p-1}\right]_{p^{n-1} \times np}$ (1)

where, M_u; $u = 0, 1, \ldots \ldots \ldots, (p-1)$ resepresent incomplete block defined as,

$M_u = [V_1\{p^0\}, V_2\{p^1\}, \ldots \ldots \ldots, V_{n-1}\{p^{n-2}\}, a_u]_{p^{n-1} \times n}$

with

$V_i\{p^{i-1}\} = p^{i-1}\left[0I_{p^{(n-1)-i}}, 1I_{p^{(n-1)-i}}, \ldots \ldots \ldots, (p-1)Ipn-1-i\ '\right.$,

each is a column vector of dimension p^{n-1}.

$\{p^{i-1}\} = p^{i-1}$ times repetitions of the elements of V_i's in ascending ordered levels.

$i = 1, 2, 3, \ldots, (n-1)$;

I_m: sum vector of dimension m; and

$a_u = \left[a_{u1}, a_{u2}, \ldots, a_{up^{n-1}}\right]'$ is called the adjustment vector.

At $u = 0$, the adjustment vector a_0 is called the key vector and resulting incomplete block represented by this key vector is called the key incomplete block. The key vector a_0 determines other adjustment vectors a_u and hence the incomplete blocks M_u, for all $u > 0$. The elements of the key vector can be obtained by solving the symbolic equation such that $\sum_i a_i F_i + a_k F_k = 0 \bmod p$ where F_k represents the adjustment factor.

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III. Limitation of The Method

This method is applicable in a p^n factorial experiment only if p is prime. When p is not a prime number, we cannot apply this method. As for example, suppose the factorial effect $F_1F_2F_3F_4$ is confounded in a 3^4 factorial experiment. In this case we can apply this method, because here $p = 3$ which is a prime number, but if we want to use this method to confound $F_1F_2F_3^2$ in 4^3 factorial experiment considering F_3 as adjustment factor, we cannot solve the following equation

$$\sum_i a_iF_i + a_kF_k = 0 \bmod p,$$

where, F_k represents the adjustment factor.

IV. Illustration I

Suppose the factorial effect $F_1F_2F_3F_4$ is to be confounded in a 3^4 factorial experiment. Here, $n = 4$; F_1, F_2, F_3 *and* F_4 ;

$p = 3;\ 0, 1$ *and* 2.

The suggested matrix is

$$M = [M_0\ M_1\ M_2]_{3^{4-1}\times 4*3} = [M_0\ M_1\ M_2]_{27\times 12}$$

where,

$$M_u = [V_1\{p^0\}, V_2\{p^1\}, \dots\dots\dots, V_{n-1}\{p^{n-2}\}, a_u]_{p^{n-1}\times n}$$

with

$$V_i\{p^{i-1}\} = p^{i-1}\left[0I_{p^{(n-1)-i}}, 1I_{p^{(n-1)-i}}, \dots\dots\dots, (p-1)I_{p^{(n-1)-i}}\right]',$$

each is a column vector of dimension p^{n-1} . Therefore, we have,

$$M_0 = [V_1\{3^0\}, V_2\{3^1\}, V_2\{3^2\}, a_0]_{3^3\times 4};$$

$$M_1 = [V_1\{3^0\}, V_2\{3^1\}, V_2\{3^2\}, a_1]_{3^3\times 4};$$

$$M_2 = [V_1\{3^0\}, V_2\{3^1\}, V_2\{3^2\}, a_2]_{3^3\times 4}$$

with

$$v_1\{1\} = 1[0I_9, 1I_9, 2I_9]'_{27\times 1},$$

$$v_2\{3\} = 3[0I_3, 1I_3, 2I_3]'_{27\times 1},$$

$$v_3\{9\} = 9[0I_1, 1I_1, 2I_1]'_{27\times 1}.$$

Now the elements of the adjustment vector are obtained from the equation $\sum_i a_iF_i + a_kF_k = 0 \bmod p$, considering $F_k = F_3$ we have,

$$v_1\{1\} = \begin{bmatrix}0\\0\\0\\0\\0\\0\\0\\0\\0\\1\\1\\1\\1\\1\\1\\1\\1\\1\\2\\2\\2\\2\\2\\2\\2\\2\\2\end{bmatrix},\ v_2\{3\} = \begin{bmatrix}0\\0\\0\\1\\1\\1\\2\\2\\2\\0\\0\\0\\1\\1\\1\\2\\2\\2\\0\\0\\0\\1\\1\\1\\2\\2\\2\end{bmatrix},\ v_3\{9\} = \begin{bmatrix}0\\1\\2\\0\\1\\2\\0\\1\\2\\0\\1\\2\\0\\1\\2\\0\\1\\2\\0\\1\\2\\0\\1\\2\\0\\1\\2\end{bmatrix} \text{ and hence, } a_0 = \begin{bmatrix}0\\2\\1\\2\\1\\0\\1\\0\\2\\2\\1\\0\\1\\0\\2\\0\\2\\1\\1\\0\\2\\0\\2\\1\\2\\1\\0\end{bmatrix} \left[\text{Solving } \sum_i a_iF_i + a_kF_k = 0 \bmod p\right]$$

Therefore,

$$M_0 = \begin{bmatrix} 0&0&0&0\\0&0&1&2\\0&0&2&1\\0&1&0&2\\0&1&1&1\\0&1&2&0\\0&2&0&1\\0&2&1&0\\0&2&2&2\\1&0&0&2\\1&0&1&1\\1&0&2&0\\1&1&0&1\\1&1&1&0\\1&1&2&2\\1&2&0&0\\1&2&1&2\\1&2&2&1\\2&0&0&1\\2&0&1&0\\2&0&2&2\\2&1&0&0\\2&1&1&2\\2&1&2&1\\2&2&0&2\\2&2&1&1\\2&2&2&0 \end{bmatrix}, M_1 = \begin{bmatrix} 0&0&0&1\\0&0&1&0\\0&0&2&2\\0&1&0&0\\0&1&1&2\\0&1&2&1\\0&2&0&2\\0&2&1&1\\0&2&2&0\\1&0&0&0\\1&0&1&2\\1&0&2&1\\1&1&0&2\\1&1&1&1\\1&1&2&0\\1&2&0&1\\1&2&1&0\\1&2&2&2\\2&0&0&2\\2&0&1&1\\2&0&2&0\\2&1&0&1\\2&1&1&0\\2&1&2&2\\2&2&0&0\\2&2&1&2\\2&2&2&1 \end{bmatrix} \text{ and } M_2 = \begin{bmatrix} 0&0&0&2\\0&0&1&1\\0&0&2&0\\0&1&0&1\\0&1&1&0\\0&1&2&2\\0&2&0&0\\0&2&1&2\\0&2&2&1\\1&0&0&1\\1&0&1&0\\1&0&2&2\\1&1&0&0\\1&1&1&2\\1&1&2&1\\1&2&0&2\\1&2&1&1\\1&2&2&0\\2&0&0&0\\2&0&1&2\\2&0&2&1\\2&1&0&2\\2&1&1&1\\2&1&2&0\\2&2&0&1\\2&2&1&0\\2&2&2&2 \end{bmatrix}$$

So, the desired plan is

$$M = \begin{bmatrix} 0&0&0&0&0&0&0&1&0&0&0&2\\0&0&1&2&0&0&1&0&0&0&1&1\\0&0&2&1&0&0&2&2&0&0&2&0\\0&1&0&2&0&1&0&0&0&1&0&1\\0&1&1&1&0&1&1&2&0&1&1&0\\0&1&2&0&0&1&2&1&0&1&2&2\\0&2&0&1&0&2&0&2&0&2&0&0\\0&2&1&0&0&2&1&1&0&2&1&2\\0&2&2&2&0&2&2&0&0&2&2&1\\1&0&0&2&1&0&0&0&1&0&0&1\\1&0&1&1&1&0&1&2&1&0&1&0\\1&0&2&0&1&0&2&1&1&0&2&2\\1&1&0&1&1&1&0&2&1&1&0&0\\1&1&1&0&1&1&1&1&1&1&1&2\\1&1&2&2&1&1&2&0&1&1&2&1\\1&2&0&0&1&2&0&1&1&2&0&2\\1&2&1&2&1&2&1&0&1&2&1&1\\1&2&2&1&1&2&2&2&1&2&2&0\\2&0&0&1&2&0&0&2&2&0&0&0\\2&0&1&0&2&0&1&1&2&0&1&2\\2&0&2&2&2&0&2&0&2&0&2&1\\2&1&0&0&2&1&0&1&2&1&0&2\\2&1&1&2&2&1&1&0&2&1&1&1\\2&1&2&1&2&1&2&2&2&1&2&0\\2&2&0&2&2&2&0&0&2&2&0&1\\2&2&1&1&2&2&1&2&2&2&1&0\\2&2&2&0&2&2&2&1&2&2&2&2 \end{bmatrix}$$

Let us consider the factorial effect $F_1F_2F_3^2$ is to be confounded in a 4^3 factorial experiment.
Here, $n = 3;\ F_1, F_2\ and\ F_3;$
$p = 4;\ 0, 1, 2\ and\ 3$
From the equation, $M = [M_0\ M_1 \dots \dots M_{p-1}]_{p^{n-1} \times np}$, we have in this case,
$M = [M_0\ \ M_1\ \ M_2\ \ M_3\ ]_{4^{3-1} \times 3*4}$
$= [M_0\ \ M_1\ \ M_2\ \ M_3\ ]_{16 \times 12};$
where,

$$M_u\ = [V_1\{p^0\}, V_2\{p^1\}, \dots \dots \dots, V_{n-1}\{p^{n-2}\}, a_u]_{p^{n-1} \times n}$$

with

$V_i\{p^{i-1}\} = p^{i-1}\left[0I_{p^{(n-1)-i}}\ ,1I_{p^{(n-1)-i}}\ ,\ldots\ldots\ldots,(p-1)I_{p^{n-1-i}}\right]'$,

each is a column vector of dimension p^{n-1} . Therefore, we have,

$M_0 = [V_1\{4^0\}, V_2\{4^1\}, a_0]_{4^2\times 3}$;
$M_1 = [V_1\{4^0\}, V_2\{4^1\}, a_1]_{4^2\times 3}$;
$M_2 = [V_1\{4^0\}, V_2\{4^1\}, a_2]_{4^2\times 3}$;
$M_3 = [V_1\{4^0\}, V_2\{4^1\}, a_3]_{4^2\times 3}$;
with
$V_1\{4^0\} = 1[0I_4\ ,1I_4\ ,2I_4\ ,3I_4\ ]'_{16\times 1}$,
$V_2\{4^1\} = 4[0I_1\ ,1I_1\ ,2I_1\ ,3I_1\ ]_{16\times 1}$.

Thus,

$$V_1\{4^0\} = \begin{bmatrix}0\\0\\0\\0\\1\\1\\1\\1\\2\\2\\2\\2\\3\\3\\3\\3\end{bmatrix},\quad V_2\{4^1\} = \begin{bmatrix}0\\1\\2\\3\\0\\1\\2\\3\\0\\1\\2\\3\\0\\1\\2\\3\end{bmatrix}$$

Now a_u is column vector of dimension 16, i.e.

$$a_u = [a_{u1}, a_{u2}, \ldots, a_{u16}]'$$

So, to get the elements of the key adjustment vector, a_0, we have to solve the following equation

$$x_1 + x_2 + 2x_3 = 0\ (mod\ 4)$$

Here we consider F_3 as adjustment factor. Thus we get,

$$0 + 0 + 2x_3 = 0\ (mod\ 4) ==> a_{01} = 0$$
$$0 + 1 + 2x_3 = 0\ (mod\ 4)$$

We cannot calculate a_{02}. Thus we cannot calculate the key adjustment vector, a_0.
The reason is discussed in the next section.

V. Reason

We know that if we multiply an even number by any natural number the result will be an even number. For this reason if we take adjustment factor, F_k, whose exponent is an even number, we cannot calculate the key adjustment vector a_0. So we have to take adjustment factor F_k whose exponent is odd number.

VI. Moderation

In this method, proposed by Jalil *et. al.*[12], we have to take that factor as adjustment factor whose exponent is odd. If there is more than one factor whose exponents are odd, then any one can be considered as adjustment factor. After moderation, this method can be applied in p^n factorial experiments, where p is prime or non prime.

VII. Illustration II

Construct a confounding plan where the confounded effect is $F_1F_2F_3^2$ in a 4^3 factorial experiment.

Solution: Here, $n = 3$; $F_1,\ F_2\ and\ F_3$;
$p = 4$; $0, 1, 2\ and\ 3$

From the equation, $M = \left[M_0\ M_1 \ldots\ldots M_{p-1}\right]_{p^{n-1}\times np}$, we have in this case,

$M = [M_0\ \ M_1\ \ M_2\ \ M_3\ ]_{4^{3-1}\times 3*4}$
$\quad = [M_0\ \ M_1\ \ M_2\ \ M_3\ ]_{16\times 12}$;
where,
$M_u = [V_1\{p^0\}, V_2\{p^1\}, \ldots\ldots\ldots, V_{n-1}\{p^{n-2}\}, a_u]_{p^{n-1}\times n}$
with
$V_i\{p^{i-1}\} = p^{i-1}\left[0I_{p^{(n-1)-i}}\ ,1I_{p^{(n-1)-i}}\ ,\ldots\ldots\ldots,(p-1)I_{p^{n-1-i}}\right]'$,

each is a column vector of dimension p^{n-1} . Therefore, we have,

$M_0 = [V_1\{4^0\}, V_2\{4^1\}, a_0]_{4^2\times 3}$;
$M_1 = [V_1\{4^0\}, V_2\{4^1\}, a_1]_{4^2\times 3}$;
$M_2 = [V_1\{4^0\}, V_2\{4^1\}, a_2]_{4^2\times 3}$;
$M_3 = [V_1\{4^0\}, V_2\{4^1\}, a_3]_{4^2\times 3}$
with
$V_1\{4^0\} = 1[0I_4\ ,1I_4\ ,2I_4\ ,3I_4\ ]'_{16\times 1}$,
$V_2\{4^1\} = 4[0I_1\ ,1I_1\ ,2I_1\ ,3I_1\ ]_{16\times 1}$.

Now the elements of the adjustment vector are obtained from the equation $\sum_i a_iF_i + a_kF_k = 0$ mod p where F_k represents the adjustment factor. Since the exponents of both $F_1\ and\ F_2$ are odd, anyone can be considered as adjustment factor. Here F_1 is considered as adjustment factor. Now, we have

$$V_1\{4^0\} = \begin{bmatrix}0\\0\\0\\0\\1\\1\\1\\1\\2\\2\\2\\2\\3\\3\\3\\3\end{bmatrix},\quad V_2\{4^1\} = \begin{bmatrix}0\\1\\2\\3\\0\\1\\2\\3\\0\\1\\2\\3\\0\\1\\2\\3\end{bmatrix}\quad \text{and hence, } a_0 = \begin{bmatrix}0\\2\\0\\2\\3\\1\\3\\1\\2\\0\\2\\0\\1\\3\\1\\3\end{bmatrix}$$

[Solving $\sum_i a_iF_i + a_kF_k = 0$ mod p].
$a_1, a_2\ and\ a_3$ can be obtained by solving $a_{uk} = a_{0k} + u$ where $k = 1, 2, \ldots, p^{n-1}$;

Therefore,

$$M_0 = \begin{bmatrix} 0 & 0 & 0 \\ 2 & 0 & 1 \\ 0 & 0 & 2 \\ 2 & 0 & 3 \\ 3 & 1 & 0 \\ 1 & 1 & 1 \\ 3 & 1 & 2 \\ 1 & 1 & 3 \\ 2 & 2 & 0 \\ 0 & 2 & 1 \\ 2 & 2 & 2 \\ 0 & 2 & 3 \\ 1 & 3 & 0 \\ 3 & 3 & 1 \\ 1 & 3 & 2 \\ 3 & 3 & 3 \end{bmatrix}, \quad M_1 = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 0 & 1 \\ 1 & 0 & 2 \\ 3 & 0 & 3 \\ 0 & 1 & 0 \\ 2 & 1 & 1 \\ 0 & 1 & 2 \\ 2 & 1 & 3 \\ 3 & 2 & 0 \\ 1 & 2 & 1 \\ 3 & 2 & 2 \\ 1 & 2 & 3 \\ 2 & 3 & 0 \\ 0 & 3 & 1 \\ 2 & 3 & 2 \\ 0 & 3 & 3 \end{bmatrix}, \quad M_2 = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & 1 \\ 2 & 0 & 2 \\ 0 & 0 & 3 \\ 1 & 1 & 0 \\ 3 & 1 & 1 \\ 1 & 1 & 2 \\ 3 & 1 & 3 \\ 0 & 2 & 0 \\ 2 & 2 & 1 \\ 0 & 2 & 2 \\ 2 & 2 & 3 \\ 3 & 3 & 0 \\ 1 & 3 & 1 \\ 3 & 3 & 2 \\ 1 & 3 & 3 \end{bmatrix}, \quad M_3 = \begin{bmatrix} 3 & 0 & 0 \\ 1 & 0 & 1 \\ 3 & 0 & 2 \\ 1 & 0 & 3 \\ 2 & 1 & 0 \\ 0 & 1 & 1 \\ 2 & 1 & 2 \\ 0 & 1 & 3 \\ 1 & 2 & 0 \\ 3 & 2 & 1 \\ 1 & 2 & 2 \\ 3 & 2 & 3 \\ 0 & 3 & 0 \\ 2 & 3 & 1 \\ 0 & 3 & 2 \\ 2 & 3 & 3 \end{bmatrix}$$

So, the desired plan is

$$M = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 2 & 0 & 0 & 3 & 0 & 0 \\ 2 & 0 & 1 & 3 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 2 & 1 & 0 & 2 & 2 & 0 & 2 & 3 & 0 & 2 \\ 2 & 0 & 3 & 3 & 0 & 3 & 0 & 0 & 3 & 1 & 0 & 3 \\ 3 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 2 & 1 & 0 \\ 1 & 1 & 1 & 2 & 1 & 1 & 3 & 1 & 1 & 0 & 1 & 1 \\ 3 & 1 & 2 & 0 & 1 & 2 & 1 & 1 & 2 & 2 & 1 & 2 \\ 1 & 1 & 3 & 2 & 1 & 3 & 3 & 1 & 3 & 0 & 1 & 3 \\ 2 & 2 & 0 & 3 & 2 & 0 & 0 & 2 & 0 & 1 & 2 & 0 \\ 0 & 2 & 1 & 1 & 2 & 1 & 2 & 2 & 1 & 3 & 2 & 1 \\ 2 & 2 & 2 & 3 & 2 & 2 & 0 & 2 & 2 & 1 & 2 & 2 \\ 0 & 2 & 3 & 1 & 2 & 3 & 2 & 2 & 3 & 3 & 2 & 3 \\ 1 & 3 & 0 & 2 & 3 & 0 & 3 & 3 & 0 & 0 & 3 & 0 \\ 3 & 3 & 1 & 0 & 3 & 1 & 1 & 3 & 1 & 2 & 3 & 1 \\ 1 & 3 & 2 & 2 & 3 & 2 & 3 & 3 & 2 & 0 & 3 & 2 \\ 3 & 3 & 3 & 0 & 3 & 3 & 1 & 3 & 3 & 2 & 3 & 3 \end{bmatrix}$$

VIII. Conclusion

The method proposed by Jalil *et. al.*[12] worked only when p is prime in p^n factorial experiments. In this article, we have made modification in taking adjustment factor, using which we can construct confounding plan with single factorial effect in p^n factorial experiments. The method is appropriate in general for any value of n, the number of factors and for any possible value of p, the levels of the factors. However, this method is restricted to p^n symmetrical factorial experiments.

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