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Application of Fuzzy Optimization Problem in Fuzzy Environment

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Abstract

In this paper application of optimization problem has been introduced which belong to fuzzy environment. An attempt has been taken to find out a suitable option in order to obtain the optimum solutions of optimization problems in fuzzy environment. Optimum solutions of the proposed optimization problem computed by using three methods, such as Bellman-Zadeh's method, Zimmerman's Method, and Fuzzy Version of Simplex method, are compared to each other. In support of that the three algorithms of the above three methods have been reviewed. However, the main objective of this paper is to focus on the appropriate method and how to achieve good enough or optimum solution of linear programming using triangular fuzzy numbers with equal widths because of complex and undefined situations in our daily life.

Keywords: Linear programming (LP), Membership function, Fuzzy number, fuzzy linear programming (FLP), Fuzzy optimization problem, Fuzzy environment, Fuzzy linear programming problem (FLPP).

I. Introduction

It has been widely observed that fuzzy set theory are often applied in the field of decision making process in fuzzy environment. To mitigate these problems researchers of various disciplines developed many mathematical models using fuzzy technique in which membership functions, introduced by Zadeh[1], have been used. As we know the uncertainty and vagueness are the essential part of our lives. So, it is impossible to eliminate all sorts of uncertainties from the living world. It also refers to those sectors or organizations, mainly dealing with manufacturing and production management. Regarding this point we would like to state that in real world decisions are taken on the basis of information which, at least in part, is fuzzy in nature. That is why fuzzy techniques are used as a useful tool to tackle those situations which cannot be handled with classical methods.

Many researchers have used fuzzy set theory to address the linear programming problems. It may be mentioned here that the linear programming model represents a real world situation, which involve many parameters whose values are assigned by the experts, which are not always realistic. In that case vagueness, uncertainty and doubtfulness cannot be denied. However, experts and decision makers frequently do not precisely know the value of these parameters[2,3]. It is, therefore, pertinent to convert linear programming into fuzzy linear programming to obtain an optimum solution when uncertainty increases in a certain problem. Fuzzy linear programming (FLP) is a refinement of linear programming (LP) which has been developed since 1970s.

The classical linear programming problem[4,5] assumes that objective and constraint are 'crisp'. The term 'to maximize' or 'to minimize' applies to the crisp case in its strictest sense. It is observed in most cases that the exact values of the constant coefficients are found either vague or imprecise because of the nature of uncertainty and imprecise information. Therefore, the constraint coefficient of the original problem needs to be replaced with fuzzy numbers that produce the fuzzy constraint of the linear programming problem which needs to be solved[6].

In fuzzy decision making problems, the concept of maximizing decision was proposed by Bellman and Zadeh[2]. Zimmermamm[7] presented a fuzzy approach to multiobjective linear programming problems. The above methods can solve the fuzzy linear programming problem which are fuzzy in nature. Moreover, those methods are not capable of solving fully fuzzy linear programming problem. Fuzzy version of simplex method is suitable to obtain an optimum solution of fully fuzzy linear programming problem.

However, in this paper we have discussed the above three methods and their limitations. An application of optimization problem, i.e., the production planning decision problem, in fuzzy environment has been introduced and focused to obtain its solution by converting the problem into fuzzy linear programming problem. Here, the production planning decision problem are related to forecasting production goal of a product, availability of the supply and their demand, which are generally fuzzy in nature. Because those variables are confronting with the real situation, for the lack of incomplete, vague or doubtful data. Also, the decision maker simultaneously handles with other conflicting situation, such as resource constraints. In particular, these conflicting situations (variables) are required to be optimized simultaneously by the decision maker. In such situation, conventional linear programming algorithm is not capable of solving production planning problems in an uncertain environment of a factory.

II. Preliminaries

Different types of methods have been evolved in order to solve the fuzzy linear programming problems[2,8]. Some related definitions are given[9].

Fuzzy Linear Programming Problem: A fuzzy linear programming problem (FLPP) is defined as follows:

$$\begin{aligned} &Maximize\ \ \tilde{Z} = \tilde{C}\tilde{X} \\ &Subject\ to\ \ \tilde{A}\tilde{X} \approx \tilde{b} \\ &\qquad\qquad \tilde{X} \geq \tilde{0} \end{aligned} \tag{1}$$

where $\tilde{A}$ is $m \times n$ matrix in fuzzy real field, i.e., $\tilde{A} \in F(\mathrm{P}^{m\times n})$; $\tilde{X}, \tilde{C}$ are $1 \times n$ matrix in fuzzy real field, i.e., $\tilde{X}, \tilde{C} \in F(\mathrm{P}^{n})$; and $\tilde{b}$ is a $m \times 1$ matrix in fuzzy real field,

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i.e., $\tilde{b} \in F(\mathrm{P}^m)$. The components of each matrix are triangular fuzzy numbers.

Triangular Membership Function: A fuzzy set $\tilde{A}$, is called triangular fuzzy number with Peak (or Core) a_2, left width $a_1 \geq 0$ and right width $a_3 \geq 0$ if its membership function has the following form

$$\mu_{\tilde{A}}(x) = \begin{cases} 1 - \dfrac{a_2 - x}{a_1} & , if\ a_2 - a_1 \leq x \leq a_2 \\ 1 - \dfrac{x - a_2}{a_3} & , if\ a_2 \leq x \leq a_2 + a_3 \\ 0 & , Otherwise. \end{cases}$$

and the set of all triangular fuzzy numbers is denoted by $\mathrm{R}(\mathcal{F})$ where $\tilde{A}$ in parametric form is

$$\tilde{A} = (a_1(r-1) + a_2, a_3(1-r) + a_2),\ r \in [0,1].$$

Symbolically, we write $\tilde{A} = (a_1, a_2, a_3)$.

Mid-point of Two Fuzzy Numbers: Let $\tilde{A}$ be a triangular fuzzy number where $\tilde{A} = (a_1, a_2, a_3)$, then, for $r \in [0,1]$, the mid-point of $\tilde{A}$ is

$$M\left(\tilde{A}(r)\right) = {}^1\!/_2\,[2a_2 + a_1(r-1) + a_3(1-r)].$$

Distance Between Two Fuzzy Numbers: Let $\tilde{A}$ and $\widetilde{\mathrm{B}}$ be two triangular fuzzy number where $\tilde{A} = (a_1, a_2, a_3), \widetilde{\mathrm{B}} = (b_1, b_2, b_3)$ and $r \in [0,1]$, then the signed distance of $\tilde{A}$ and $\tilde{B}$ are as follows:

$$d(\tilde{A}, \tilde{B}) = \int_0^1 [M\left(\tilde{A}(r)\right) - M\left(\widetilde{\mathrm{B}}(r)\right)]dr$$
$$= \frac{1}{4}[4(a_2 - b_2) - (a_1 - b_1) + (a_3 - b_3)].$$

Zero Triangular Fuzzy Number: A triangular fuzzy number $\tilde{A} = (a_1, a_2, a_3)$ is said to be zero triangular fuzzy number if and only if $4a_2 - a_1 + a_3 = 0$, or a triangular fuzzy number $\tilde{A}$ is said to be zero triangular fuzzy number if $Core(\tilde{A}) = 0$. It is to be noted that $\tilde{0}$ is equivalent to $(0,0,0)$ and the signed distance of triangular fuzzy number $\tilde{A} = (a_1, a_2, a_3)$ is $d(\tilde{A}, \tilde{0}) = \frac{1}{4}[4a_2 - a_1 + a_3]$.

Operations on Triangular Fuzzy Number: Let $\tilde{A}$ and $\tilde{B}$ be two triangular fuzzy numbers where

$\tilde{A} = (a_1, a_2, a_3), \widetilde{\mathrm{B}} = (b_1, b_2, b_3)$. Then

(i) $\tilde{A} + \widetilde{\mathrm{B}} = (a_1, a_2, a_3) + (b_1, b_2, b_3) = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$;

(ii) $-\widetilde{\mathrm{B}} = -(b_1, b_2, b_3) = (b_3, -b_2, b_1)$;

(iii) $\tilde{A} - \widetilde{\mathrm{B}} = (a_1, a_2, a_3) - (b_1, b_2, b_3) = (a_1 + b_3, a_2 - b_2, a_3 + b_1)$;

(iv) $\tilde{A}.\widetilde{\mathrm{B}} = (a_1, a_2, a_3).(b_1, b_2, b_3) = (a_2 b_1 + b_2 a_1, a_2 b_2, a_2 b_3 + b_2 a_3)$, if $\tilde{A}$ and $\tilde{B}$ are positive;

(v) $\tilde{A}.\widetilde{\mathrm{B}} = (a_3, -a_2, a_1).(b_1, b_2, b_3) = (b_2 a_3 - a_2 b_1, -a_2 b_2, b_2 a_1 - a_2 b_3)$, if $\tilde{A}$ is negative and $\tilde{B}$ is positive;

(vi) $\tilde{A}.\widetilde{\mathrm{B}} = (a_3, -a_2, a_1).(b_3, -b_2, b_1) = (-a_2 b_3 - b_2 a_3, a_2 b_2, -a_2 b_1 - b_2 a_1)$, if $\tilde{A}$ and $\tilde{B}$ are negative;

(vii) $A^{-1} = 1/\tilde{A} = (1/a_1, 1/a_2, 1/a_3)$;

(viii) $\tilde{A}/\widetilde{\mathrm{B}} = (a_1, a_2, a_3)/(b_1, b_2, b_3) = (a_2/b_1 + a_1/b_2, a_2/b_2, a_2 b_3 + a_3/b_2)$, if $\tilde{A}$ and $\tilde{B}$ are positive;

(ix) $\tilde{A}/\widetilde{\mathrm{B}} = (a_3, -a_2, a_1).(b_1, b_2, b_3) = (a_3/b_2 - a_2/b_1, -a_2/b_2, a_1/b_2 - a_2/b_3)$, if $\tilde{A}$ is negative and $\tilde{B}$ is positive;

(x) $\tilde{A}/\widetilde{\mathrm{B}} = (a_3, -a_2, a_1)/(b_3, -b_2, b_1) = (-a_2/b_3 - a_3/b_2, a_2/b_2, -a_2/b_1 - a_1/b_2)$, if $\tilde{A}$ and $\tilde{B}$ are negative;

(xi) $\lambda\tilde{A} = \begin{cases} (\lambda a_1, \lambda a_2, \lambda a_3) & , if\ \lambda > 0 \\ (-\lambda a_1, -\lambda a_2, -\lambda a_3) & if\ \lambda > 0 \end{cases}$.

III. Fuzzy Linear Programming (FLP)

There are three types of fuzzy linear programming which are discussed below:

Type 1: A fuzzy linear programming, in which only the right hand side constraints B_i are fuzzy numbers, is as follows:

$$\begin{aligned} & max \textstyle\sum_{j=1}^{n} c_j x_j \\ & s.t. \textstyle\sum_{j=1}^{n} a_{ij} x_j \leq B_i \quad (i \in N_m) \\ & x_j \geq 0 \quad (j \in N_n) \end{aligned} \tag{2}$$

This type of FLP can be solved by the method of Bellman-Zadeh. The algorithm to obtain the solution of FLP by Bellman-Zadeh's method is as under[3] :

Step 1: Suppose that the right-hand side constraints are fuzzy numbers B_i where the membership function $B_i(x)$ are defined as

$$B_i(x) = \begin{cases} 1, & when\ x \leq b_i \\ \frac{b_i + p_i - x}{p_i}, & when\ b_i \leq x \leq b_i + p_i \\ 0, & when\ x \geq b_i + p_i \end{cases}.$$

Step 2: Calculate the lower bounds of the objective function z_l.

Step 3: Calculate the upper bounds of the objective function z_u.

Step 4: Define the fuzzy set G of optimal values, as follows:

$$G(z) = \begin{cases} 1, & when\ z_u \leq z \\ \frac{(z - z_l)}{z_u - z_l}, & when\ z_l \leq z \leq z_u \\ 0, & when\ z \leq z_l \end{cases}.$$

Step 5: Introduce a new variable λ which denotes the membership grade of the solution and also introduce a new constraints equation is $\lambda(z_u - z_l) - z \leq -z_l$.

Step 6: Convert the fuzzy linear programming problem into equivalent crisp linear programming problem as follows:

$$Max\ \lambda$$
$$s.t.\quad \lambda(z_u - z_l) - z \le -z_l$$
$$\lambda p_i + \sum_{i=1}^{n} a_{ij}x_j \le b_i + p_i \quad (i \in N_m)$$
$$\lambda, x_j \ge 0 \quad (j \in N_n)\ .$$

Step 7: Solve it for the variables x_j and λ where the new variable λ denote the membership grade of the solution using Computer Assisted Software (CAS).

Step 8: Put the values of x_j into the objective function so that we can get the optimal solution.

Type 2: A fuzzy linear programming in which only the right hand side constraints B_i and the objective function are fuzzy numbers, but the coefficients A_{ij} of the constant matrix are not fuzzy numbers, is:

$$max \sum_{j=1}^{n} c_j x_j$$
$$s.t. \sum_{j=1}^{n} A_{ij}x_j \le B_i \quad (i \in N_m) \tag{3}$$
$$x_j \ge 0 \quad (j \in N_n)$$

In model (1) we assume that the decision maker can establish an aspiration level, z, for the value of the objective function which he desires to achieve and then each of the constraints is modeled as a fuzzy set. Then the fuzzy LP stands:

Find x such that

$$C^T x \gtrsim z$$
$$Ax \lesssim b \tag{3a}$$
$$x \ge 0$$

where $\gtrsim$ denotes the fuzzified version of $\ge$ and $\lesssim$ denotes the fuzzified version of $\le$. This type of FLP is solved by using the algorithm of Zimmerman's Method and the algorithm is given below[8] :

Step 1: We define the matrix A and b such that

$$Bx \lesssim d$$
$$x \ge 0\ ,$$

where $\binom{-C^T}{A} = B$ and $\binom{-Z}{b} = d$.

Step 2: Estimate lower bound of tolerance intervals d_i.

Step 3: Spread the tolerance intervals p_i.

We define membership function μ_i as follows:

$$\mu_i(x) = \begin{cases} 1, & when\ B_i x \le d_i \\ 1 - \frac{B_i x - d_i}{p_i} & when\ d_i < B_i x \le d_i + p_i, \quad i = 1,2,\ldots\ldots,m. \\ 0, & when\ B_i x > d_i + p_i \end{cases}$$

Step 4: Introduce a new variable λ and calculate

$$\frac{\lambda\, p_i + B_i x}{p_i} \le \frac{d_i + p_i}{p_i}\ , \quad i = 1,2,\ldots\ldots,m+1.$$

Step 5: We convert the fuzzy linear programming into equivalent crisp linear programming problem as follows:

$$max \quad \lambda$$
$$\lambda\, p_i + B_i x \le d_i + p_i\ , \quad i = 1,2,\ldots\ldots m+1$$
$$x \ge 0$$

where $x = (x_j),\ \ j \in N_n$.

Step 6: We solve it for the variables x and λ where the new variable λ denote the membership grade of the solution and we get the solution using CAS.

Step 7: Put the values of x_j into the objective function so that we can get the optimal solution.

Type 3: A fuzzy linear programming, in which all parameters and variables are considered as triangular fuzzy numbers, is formulated as follows:

$$max\ z \approx \sum_{j=1}^{n} C_j X_j$$
$$s.t. \sum_{j=1}^{n} A_{ij} X_j \preccurlyeq B_i \quad (i \in N_m) \tag{4}$$
$$X_j \succcurlyeq 0 \quad (j \in N_n)$$

where A_{ij}, B_i, C_j are fuzzy numbers and X_j are variables whose states are fuzzy numbers $(i \in N_m, j \in N_n)$; the operations of additions and multiplications are operations of fuzzy arithmetic; and $\preccurlyeq$, $\succcurlyeq$ denote the ordering of fuzzy numbers. Instead of discussing this general type, we exemplify the issues involved by two special cases of fuzzy linear programming problems.

The Fuzzy Version of Simplex Algorithm is used to solve this type of FLP. Here we are going to discuss the fuzzy version of simplex method. For the solution of any FLPP by Simplex algorithm, the existence of an initial basic feasible fuzzy solution is always assumed. The steps of the fuzzy version of Simplex algorithm for the computation of an optimum fuzzy solution are as follows[2] :

Step 1: Check whether the objective function of the given FLP is to be minimized or maximized. If it is to be minimized then we convert it into a problem of maximizing it by using the result

$$Minimum\ \tilde{z} = -Maximum(-\tilde{z}).$$

Step 2: Check whether all $\tilde{b}_i (i = 1,2,\ldots\ldots,m)$ are non-negative. If any one of $\tilde{b}_i$ is negative then multiply the corresponding inequation of the constant by -1, so as to get all $\tilde{b}_i (i = 1,2,\ldots\ldots,m)$ non-negative.

Step 3: Convert all the inequalities of the constraints into equations by introducing slack and / or surplus fuzzy variables in the constraints. Put the costs of these variables equal to zero.

Step 4: Obtain an initial basic feasible solution to the problem in the problem in the form $\tilde{X}_B \approx \tilde{B}^{-1}\tilde{b}$ and put in the first column of the Simplex table.

Step 5: Compute the net evaluations $\tilde{z}_j - \tilde{c}_j\,(j = 1,2,\ldots\ldots,n)$ by using the relation $\tilde{z}_j - \tilde{c}_j = \tilde{c}_B\tilde{y}_j - c_j$, where any column $\tilde{A}_j \approx \sum_{i=1}^{m} \tilde{y}_{ij}\tilde{b}_i \approx \tilde{y}_{1j}\tilde{b}_1 + \tilde{y}_{2j}\tilde{b}_2 + \cdots + \tilde{y}_{rj}\tilde{b}_r + \cdots + \tilde{y}_{mj}\tilde{b}_m \approx \tilde{y}_j\tilde{B}$.

Examine the sign of $\tilde{z}_j - \tilde{c}_j$:

(i) If all $(\widetilde{z}_j - \tilde{c}_j) \succcurlyeq \tilde{0}$ then the initial basic feasible fuzzy solution $\tilde{x}_B$ is an optimum basic feasible fuzzy solution.

(ii) If at least one $(\widetilde{z}_j - \tilde{c}_j) \prec \tilde{0}$, proceed on to the next step.

Step 6: If there are more than one negative $\tilde{z}_j - \tilde{c}_j$, then choose the most negative of them. Let it be $\tilde{z}_r - \tilde{c}_r$ for some $j = r$.

(i) If all $\tilde{y}_{ir} \preccurlyeq \tilde{0}(i = 1,2, \dots \dots, m)$, then there is an unbounded solution to the given problem.

(ii) If at least one $\tilde{y}_{ir} \succ \tilde{0}(i = 1,2, \dots \dots, m)$, then the corresponding vector $\tilde{y}_r$ enter the basis $\tilde{y}_B$.

Step 7: Compute $\frac{\tilde{x}_{Bi}}{\tilde{y}_{ir}}, i = 1,2, \dots \dots., m$ and choose minimum of them. Let minimum of these ratios be ${}^{\tilde{x}_{Br}}/_{\tilde{y}_{kr}}$. Then the vector $\tilde{y}_k$ will level the basis $\tilde{y}_B$. The common element $\tilde{y}_{kr}$, which in the k^{th} row and r^{th} column is known as leading triangular fuzzy number of the table.

Step 8: Convert the leading triangular fuzzy number to unit triangular fuzzy number by dividing its row by the leading triangular fuzzy number itself and all other elements in its column to zero triangular fuzzy number by making use of relation:

$$\hat{\tilde{y}} \approx \tilde{y}_{ij} - (\tilde{y}_{kj} / \tilde{y}_{kr})\tilde{y}_{ir} \;, i = 1,2, \dots \dots m + 1; i \neq k$$

and $\tilde{y}_{kj} \approx \tilde{y}_{kj} / \tilde{y}_{kr} \;, j = 0,1,2, \dots \dots, n.$

Step 9: Go to step 5 and repeat the computational procedure until either an optimum solution is obtained or these is an indication of an unbounded solution.

IV. Comparison Among Bellman-Zadeh's Method, Zimmermann's Method and Fuzzy Version of Simplex Method

Bellman-Zadeh's method is used to solve such types of fuzzy linear programming problems whose right hand side constraints are considered as fuzzy numbers. On the other hand, Zimmerman's method is used to solve the linear programming problems whose objective- functions and the constraints B_i are fuzzy numbers. This method is incapable of solving the FLPP whose right hand side constraints B_i are fuzzy numbers. It is noted that Zimmerman's method is also capable of solving the FLPP which can be solved by Bellman-Zadeh's method. So, this is the basic advantage of the Zimmerman's method. In this respect it can be said that Zimmerman's method is superior than Bellman-Zadeh's method. It may be mentioned that in both methods fuzzy linear programming problems needs to be converted into equivalent crisp linear programming problems and then the problems are solved by standard classical methods. It is worth to mention here that the final results of the fuzzy linear programming problems would be real numbers.

Fuzzy simplex method is used to solve fuzzy linear programming problems in which all parameters and variables are considered as triangular fuzzy numbers. This types of linear programming are called fully fuzzy linear programming. Zimmerman's method and Bellman-Zadeh's method are not able to solve fully fuzzy linear programming problems discussed above. Here it is noted that Fuzzy version of simplex method is able to solve the two types of fuzzy linear programming problems discussed above. So, it is concluded that the Fuzzy version of simplex method is most efficient method among the other known methods.

V. An Application of Optimization Problems

Fuzzy linear programming has great importance in production planning problem in a factory or a production oriented company. We may consider that each and every company has the aspiration to achieve or optimize more profit using minimum input. That is why they make a goal to achieve the highest margin of revenue or profit. For doing so they set an annual production plan considering their business constraints (variables). In fact in our society or in the environment we are living everything is uncertain. So we need to formulate company's production plan. This can be set by using fuzzy linear programming. An application is proposed in the following section in favour of the above arguments.

The Production Planning Problem

Problem : A Production Planning problem is stated as :

A certain company has a factory, which produces 2 products, such as, products A and B. To produce 1 ton of product A it needs 3 ton of material X and 5 ton of material Y. To produce 1 ton of product B it needs 5 ton of material X and 2 ton of material Y. The available amount of material X varies from 15 to 20 ton and Y varies from 10 to 15 ton. At the current price, the company expects to sell product A at a rate of 5 million/ton, product B at a rate of 3 million/ton. The object of the company is to maximize total revenue.

Solution: First we formulate the problem as a linear programming problem when the available amount of materials X and Y are fixed, say 15 ton and 10 ton respectively. Let x and y be the amount of product A and product B respectively which the company produces. So, x and y are the unknown variables to be determined (decision variables). Here, $x \geq 0, \; y \geq 0$.

The available amount of material X is 15 ton. So we have $3x + 5y \leq 15$. Similarly, The available amount of material Y is 10 ton. So, $5x + 2y \leq 10$. Therefore, the total expected profit is $5x + 3y$. So, the objective function is $z = 5x + 3y$ which is to be maximized.

The formulation of LP is given below:

$$\begin{aligned} &Max \quad z = 5x + 3y \\ &s.t. \quad 3x + 5y \leq 15 \\ &\qquad 5x + 2y \leq 10 \\ &\qquad x, y \geq 0 \,. \end{aligned} \tag{5}$$

Solving the above linear programming problem using classical simplex algorithm we have,

$$x = \frac{20}{19} = 1.05263,\ y = \frac{45}{19} = 2.36842$$

and $z = 5 \times 20/19 + 3 \times 45/19 = 235/19 = 12.3684$.

So, we conclude that the company's maximum revenue is 12.3684 million where the production of A is $x = 1.05263$ ton and the production of B is $y = 2.36842$ ton.

Solution using Bellman-Zadeh's Algorithm: Let us consider the linear programming problem given in (5) and (2). We construct the following FLPP in which the right hand side constraints B_1 varies from 15 ton to 20 ton and amount of material B_2 varies from 10 ton to 15 ton are fuzzy numbers

$$max\, z = 5x + 3y$$
$$s.t. 3x + 5y \leq B_1$$
$$5x + 2y \leq B_2$$
$$x, y, \geq 0$$

where the membership functions are defined as

$$B_1(x) = \begin{cases} 1, & when\ x \leq 15 \\ \frac{20-x}{5}, & when\ 15 \leq x \leq 20 \\ 0, & when\ x \geq 20 \end{cases}$$

and $$B_2(x) = \begin{cases} 1, & when\ x \leq 10 \\ \frac{15-x}{5}, & when\ 10 \leq x \leq 15 \\ 0, & when\ x \geq 15 \end{cases}.$$

The solution of the fuzzy linear programming problem by using Bellman-Zadeh's Algorithm is as under:

```
Input: Maximize[{λ,5.5263λ-5x3y≤12.3684,3x+5y+5λ≤20,5x+2y+5λ≤15,x≥0,y≥0,λ≥0},{x,y,λ}]
```

```
Output:{0.500003,{x→1.44737,y→2.63158,λ→0.500003}}.
```

Putting the value of x and y in the objective function, we have z = 5 × 1.44737 + 3 × 2.63158 = 15.13159.

We observe that the company's maximum revenue is $z = 15.13159$ million where production of A is $x = 1.44737$ ton and production of B is $y = 2.63158$ ton.

Solution using Zimmerman's Algorithm: Consider the linear programming problem discussed in (5) and (3).

We first solve (5), and get $x = 1.05263$, $y = 2.36842$, $z_l = 12.3684$ with the help of classical simplex algorithm. Then, we consider the fuzzy version of the LP (5) which is of the form (3). Now, we need to solve this FLPP by using the algorithm of type 2.

We define the matrix $A = \begin{pmatrix} 3 & 5 \\ 5 & 2 \end{pmatrix}$ and $b = \begin{pmatrix} 15 \\ 10 \end{pmatrix}$ such that

$$Bx \gtrsim d$$
$$x \geq 0$$

where $\begin{pmatrix} -5 & -3 \\ 3 & 5 \\ 5 & 2 \end{pmatrix} = \binom{-cT}{A} = B; \begin{pmatrix} -12.3684 \\ 15 \\ 10 \end{pmatrix} = \binom{-z}{b} = d.$

Now we estimate lower bound of tolerance intervals d_i, that is, $d_1 = 12.3684$, $d_2 = 15$, $d_3 = 10$.

We spread the tolerance intervals p_i, i. e., $p_1 = 5$, $p_2 = 5$, $p_3 = 5$. Introduce a new variable λ and calculate

$$\frac{\lambda\, p_1 + B_1 x}{p_1} \leq \frac{d_1 + p_1}{p_1},\ \frac{\lambda\, p_2 + B_2 x}{p_2} \leq \frac{d_2 + p_2}{p_2}$$

and $\frac{\lambda\, p_3 + B_3 x}{p_3} \leq \frac{d_3 + p_3}{p_3}$.

So, we have

$$-\,^5/_5\, x - \,^3/_5\, y + \lambda \leq \frac{-12.3684+5}{5};$$
$$^3/_5\, x + \,^5/_5\, y + \lambda \leq (15+5)/5;$$
$$^5/_5\, x + \,^2/_5\, y + \lambda \leq (10+5)/5.$$

Then, we convert the fuzzy linear programming into equivalent crisp problem as follows:

$$max\, \lambda$$
$$s.t. -x - \,^3/_5\, y + \lambda \leq -1.47368$$
$$^3/_5\, x + \,^5/_5\, y + \lambda \leq 4$$
$$^5/_5\, x + \,^2/_5\, y + \lambda \leq 3$$
$$\lambda, x, y \geq 0.$$

We solve it for the variables x, y and λ where the new variable λ denote the membership grade of the solution using CAS. The command and the solution are as follows:

```
Input: Maximize[{λ,-x-0.6 y+λ≤(-12.3684+5)/5, 0.6 x+y+λ≤4, x+0.4 y+λ≤3},{x,y,λ}]//N.
```

```
Output: {1.0, {x→1.05263,y→2.36842,λ→1.}}.
```

Now, putting the values of x, y into the objective function, we obtain

$$z = 5 \times 1.05263 + 3 \times 2.36842 = 12.3684.$$

Therefore, we conclude that the company's maximum revenue is $z = 12.3684$ million where production of A is $x = 1.05263$ ton and production of B is $y = 2.36842$ ton.

Solution Using Fuzzy Version of Simplex Algorithm: Consider the FLP which is the fuzzy form of the classical LP (5) where all the parameters and variables are fuzzy numbers:

$$Maximizing\ \tilde{Z} \approx (1,5,2)\tilde{x}_1 + (1,3,1)\tilde{x}_2$$
$$(2,3,1)\tilde{x}_1 + (3,5,1)\tilde{x}_2 \preccurlyeq (5,15,8)$$
$$(1,5,2)\tilde{x}_1 + (.5,2,1)\tilde{x}_2 \preccurlyeq (6,10,3) \quad (6)$$
$$\tilde{x}_1, \tilde{x}_2 \succcurlyeq \tilde{0}$$

For the solution of any FLPP by fuzzy version of Simplex algorithm, the existence of an initial basic feasible fuzzy solution is always assumed. First of all we convert all the inequalities of the constraints into equations by introducing slack and / or surplus fuzzy variables in the constraints and put the costs of these variables equal to zero to convert the problem to standard form as follows:

$Maximizing\ \tilde{Z} \approx (1,5,2)\tilde{x}_1 + (1,3,1)\tilde{x}_2 + (0,0,0)\tilde{x}_3 + (0,0,0)\tilde{x}_4$

$(2,3,1)\tilde{x}_1 + (3,5,1)\tilde{x}_2 + (0,1,0)\tilde{x}_3 \approx (5,15,8)$ $(1,5,2)\tilde{x}_1 + (.5,2,1)\tilde{x}_2 + +(0,1,0)\tilde{x}_4 \approx (6,10,3)\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tilde{x}_4 \succcurlyeq \tilde{0}.$

Now, we follow the algorithm of Type 3 and obtain the following tables.

Table 1. Simplex Table

			(1,5,2)	(1,3,1)	(0,0,0)	(0,0,0)	
$\tilde{c}_B$	$\tilde{y}_B$	$\tilde{x}_{Bi}$	$\tilde{y}_1$	$\tilde{y}_2$	$\tilde{y}_3$	$\tilde{y}_4$	$\frac{\tilde{x}_{Bi}}{\tilde{y}_{i1}}$
(0,0,0)	$\tilde{y}_3$	(5,15,8)	(2,3,1)	(3,5,1)	(0,1,0)	(0,0,0)	$\left(\frac{55}{6}, 5, \frac{53}{3}\right)$
(0,0,0)	$\tilde{y}_4$	(6,10,3)	$(1,5,2)^*$	(.5,2,1)	(0,0,0)	(0,1,0)	$\left(\frac{56}{5}, 2, \frac{28}{5}\right)$
	$(\tilde{z}_j - \tilde{c}_j)$	(0,0,0)	(2, −5,1)	(1, −3,1)	(0,0,0)	(0,0,0)	

Table 2. Simplex Table

			(1,5,2)	(1,3,1)	(0,0,0)	(0,0,0)	
$\tilde{c}_B$	$\tilde{y}_B$	$\tilde{x}_{Bi}$	$\tilde{y}_1$	$\tilde{y}_2$	$\tilde{y}_3$	$\tilde{y}_4$	$\frac{\tilde{x}_{Bi}}{\tilde{y}_{i2}}$
(0,0,0)	$\tilde{y}_3$	$\left(\frac{119}{5}, 9, \frac{228}{5}\right)$	$\left(\frac{117}{10}, 0, \frac{93}{5}\right)$	$\left(7, \frac{19}{5}, \frac{81}{10}\right)^*$	(0,1,0)	$\left(\frac{17}{10}, -\frac{3}{5}, \frac{17}{5}\right)$	$\left(\frac{1004}{133}, \frac{45}{19}, \frac{118}{9}\right)$
(1,5,2)	$\tilde{y}_1$	$\left(\frac{56}{5}, 2, \frac{28}{5}\right)$	$\left(\frac{26}{5}, 1, \frac{29}{10}\right)$	$\left(\frac{21}{10}, \frac{2}{5}, \frac{6}{5}\right)$	(0,0,0)	$\left(1, \frac{1}{5}, \frac{1}{2}\right)$	$\left(\frac{608}{21}, 5, \frac{47}{3}\right)$
	$(\tilde{z}_j - \tilde{c}_j)$	(58,10,32)	$\left(29, 0, \frac{35}{2}\right)$	$\left(\frac{119}{10}, -1, \frac{39}{5}\right)$	(0,0,0)	$\left(\frac{26}{5}, 1, \frac{29}{10}\right)$	

Table 3. Simplex Table

			(1,5,2)	(1,3,1)	(0,0,0)	(0,0,0)
$\tilde{c}_B$	$\tilde{y}_B$	$\tilde{x}_B$	$\tilde{y}_1$	$\tilde{y}_2$	$\tilde{y}_3$	$\tilde{y}_4$
(1,3,1)	$\tilde{y}_2$	$\left(\frac{1004}{133}, \frac{45}{19}, \frac{118}{9}\right)$	$\left(\frac{117}{38}, 0, \frac{93}{19}\right)$	$\left(\frac{1586}{665}, 1, \frac{8005}{3078}\right)$	$\left(\frac{1}{7}, \frac{5}{19}, \frac{10}{81}\right)$	$\left(\frac{709}{1330}, -\frac{3}{19}, \frac{497}{513}\right)$
(1,5,2)	$\tilde{y}_1$	$\left(\frac{3298}{171}, \frac{20}{19}, \frac{18079}{1330}\right)$	$\left(\frac{136}{19}, 1, \frac{157}{38}\right)$	$\left(\frac{66797}{15390}, 0, \frac{28289}{6650}\right)$	$\left(\frac{562}{1539}, -\frac{2}{19}, \frac{811}{1330}\right)$	$\left(\frac{809}{513}, \frac{5}{19}, \frac{3474}{3325}\right)$
	$(\tilde{z}_j - \tilde{c}_j)$	$\left(\frac{146633}{1197}, \frac{235}{19}, \frac{89195}{798}\right)$	$\left(\frac{1825}{38}, 0, \frac{1457}{38}\right)$	$\left(\frac{62460119}{201609010}, 0, \frac{557899}{17955}\right)$	$\left(\frac{28256}{10773}, \frac{5}{19}, \frac{87019}{68229}\right)$	$\left(\frac{18721}{1890}, \frac{16}{19}, \frac{52756}{5985}\right)$

From the Table 3 we obtain the optimum solution of the problem (6) which is given below:

$$\tilde{Z} = \left(\frac{146633}{1197}, \frac{235}{19}, \frac{89195}{798}\right)$$

with $\tilde{y}_1 = \left(\frac{3298}{171}, \frac{20}{19}, \frac{18079}{1330}\right)$, and $\tilde{y}_2 = \left(\frac{1004}{133}, \frac{45}{19}, \frac{118}{9}\right)$.

So, we conclude that the company's maximum revenue is $\tilde{Z} = \left(\frac{146633}{1197}, \frac{235}{19}, \frac{89195}{798}\right)$ million,

Table 4. Problems and Their Solutions

No.	Problem	Solution
1.	$Max\ \tilde{Z} \approx (1,5,2)\tilde{y}_1 + (1,3,1)\tilde{y}_2$ $(1,3,2)\tilde{y}_1 + (3,5,1)\tilde{y}_2 \preccurlyeq (5,15,8)$ $(1,5,2)\tilde{y}_1 + (.5,2,1)\tilde{y}_2 \preccurlyeq (6,10,3)$ $\tilde{y}_1, \tilde{y}_2 \succcurlyeq \tilde{0}$	$\tilde{Z} = 9.64286$ with $\tilde{y}_1 = 0$, $\tilde{y}_2 = 1.67857$
2.	$Max\ \tilde{Z} \approx (2,5,2)\tilde{y}_1 + (1,3,1)\tilde{y}_2$ $(2,3,2)\tilde{y}_1 + (3,5,3)\tilde{y}_2 \preccurlyeq (5,15,5)$ $(2,5,2)\tilde{y}_1 + (1,2,1)\tilde{y}_2 \preccurlyeq (3,10,3)$ $\tilde{y}_1, \tilde{y}_2 \succcurlyeq \tilde{0}$	$\tilde{Z} = 12.3684$ with $\tilde{y}_1 = 1.05263$, $\tilde{y}_2 = 2.36842$
3.	$Max\ \tilde{Z} \approx (2,5,2)\tilde{y}_1 + (1,3,1)\tilde{y}_2$ $(2,3,2)\tilde{y}_1 + (3,5,3)\tilde{y}_2 \preccurlyeq (5,15,5)$ $(2,5,2)\tilde{y}_1 + (1,2,1)\tilde{y}_2 \preccurlyeq (6,10,6)$ $\tilde{y}_1, \tilde{y}_2 \succcurlyeq \tilde{0}$	$\tilde{Z} = 11.0526$ with $\tilde{y}_1 = 1.57895, \tilde{y}_2 = 1.05263$
4.	$Max\ \tilde{Z} \approx (2,5,2)\tilde{y}_1 + (1,3,1)\tilde{y}_2$ $(2,3,2)\tilde{y}_1 + (3,5,3)\tilde{y}_2 \preccurlyeq (2,15,2)$ $(2,5,2)\tilde{y}_1 + (1,2,1)\tilde{y}_2 \preccurlyeq (1,10,1)$ $\tilde{y}_1, \tilde{y}_2 \succcurlyeq \tilde{0}$	$\tilde{Z} = 12.3684$ with $\tilde{y}_1 = 1.05263, \tilde{y}_2 = 2.36842$

where production of A is $\tilde{y}_1 = \left(\frac{3298}{171}, \frac{20}{19}, \frac{18079}{1330}\right)$ ton and production of B is$\tilde{y}_2 = \left(\frac{1004}{133}, \frac{45}{19}, \frac{118}{9}\right)$ ton.

Defuzzifying the maximum revenue we have $z = \frac{1}{4}\left(4 \times \frac{235}{19} - \frac{146633}{1197} + \frac{89195}{798}\right) = 9.68661$ million.

Here it is noted that the problem (6) is formulated by using the triangular fuzzy number with left width a_1 and right width a_3 of the form $\tilde{A} = (a_1, a_2, a_3)$ where $a_1 \neq a_3$. Now, we formulate the same problem by using different triangular fuzzy numbers with $a_1 \neq a_3$ or $a_1 = a_3$. The problems and their results are shown in Table 4.

We have already observed in section IV that the fuzzy version of simplex algorithm is most suitable among three methods discussed in section III. From the results of the problems, shown in the Table 4, it is observed that if we use triangular fuzzy number $\tilde{A} = (a_1, a_2, a_3)$ with equal widths, that is, $a_1 = a_3$, then the solutions are more acceptable than the solutions by using triangular fuzzy numbers of unequal width, i.e., $a_1 \neq a_3$, which is a new contribution to this paper.

Now we may say, if the uncertainty of the supply of the raw materials used in a factory increases, then the revenue of a certain company will decrease simultaneously.

VI. Conclusion

In this paper, the fuzzy linear programming problem has been discussed. An example of fuzzy optimization problem has been modeled, which works as fully fuzzy linear programming problem by using triangular fuzzy numbers and solved it with the help of fuzzy version of simplex algorithm. Whereas Bellman-Zadeh's algorithm and Zimmerman's algorithm is not capable of solving the same problem.

Same fuzzy optimization problem has been modeled by considering different fuzzy linear programming problems, shown in Table 4, by using triangular fuzzy numbers with different widths and solved them with the help of fuzzy version of simplex algorithm. From Table 4, it is observed that more acceptable, that is, good enough solutions yield if triangular fuzzy numbers of the form $\tilde{A} = (a_1, a_2, a_3)$ with equal widths, that is, $a_1 = a_3$, are used. So, the triangular fuzzy numbers with equal widths need to be used in the fuzzy linear programming problem to obtain a good enough solution of a production planning problem.

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