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Matrix Method of Designing Simultaneous Confounding in a 5^n - Factorial Experiment

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Abstract

This article describes matrix method of designing simultaneous confounding of two factorial effects in a 5^n - factorial experiment. It becomes easier to construct the design of simultaneous confounding of two factors in a 5^n- factorial experiment especially when the number of factors as well as the number of levels becomes larger.

I. Introduction

The practical works in the construction of confounding plan with factorial experiment become troublesome especially when the number of factors as well as the number of levels of each of the factors is large. This trouble becomes more difficult if we have no required number of homogeneous plots in practice. In such situation, we are bound to use a limited number of homogeneous plots to analyze the factorial effects. As a result, some factorial effects or interactions will be mixed up with block effect, i.e. confounded. Since there is no way to avoid this, the higher order interaction effects are considered to be confounded. We usually consider one or two higher order interaction effects to be confounded to perform analysis efficiently.

A system of simultaneous confounding in 2^n factorial experiment has been described, where an intrablock subgroup is constructed with the common elements taken from the factorial effects of two incomplete blocks, each confounded with a single factorial effect (Kempthorne, 1947, 1952)[1,2]. All the treatment combinations were taken into account in a manipulating but systematic manner. Mallick, S. A. (1973 & 1975)[3, 4] developed two systems of designing factorial effects with simultaneous confounding of two effects, one for a 3^n and the other for a 4^n - factorial experiments. In all these systems of simultaneous confounding, the design of factorial effects were on some manipulating manner based on solving the symbolic equations used in confounding ph factorial experiment. Jalil, M. A. et. al. (1990) developed matrix method of designing a single factorial effect confounded in a p^n - factorial experiment[5], where the level combinations are obtained by matrix operations of the levels. The present work is also a matrix method of designing a 5^n- factorial experiment of simultaneous confounding of two factorial effects.

Method

In the construction of a p^n - factorial experiment (p is prime number) with a single factorial confounded, we can write the level combinations by the equation described below (Jalil, M. A. et al 1990).

$$M = [M_0, M_1, M_2, \cdots, M_{p-1}]; \qquad (1)$$

where incomplete blocks,

$M_u;\ u = 0,1,2,\cdots,(p-1)$ is given by:

$M_u = [V_1\{p^0\}, V_2\{p^1\}, \cdots, V_{n-1}\{p^{p-1}\}, a_u]_{p^{n-1}\times n}$ with

$V_i\{p^j\} = p^j[0\, I_{p^{(n-1)-i}}, 0\, I_{p^{(n-1)-i}}, \cdots, (p-1)\, I_{p^{(n-1)-i}}]$

each is a column vector of dimension p^n.

$u = 0,1,2,\cdots,(p-1);\ i = 1,2,3,\cdots,(n-1);$

$j = 0,1,2,\cdots,(p-1);$ with the restriction that $i = j+1$;

I_m : sum vector of dimension m ;

$\{p^j\} = p^j -$ times repetition of the elements of V_i in ascending ordered levels; and

$a_u = [a_{u1}, a_{u2}, \cdots, a_{up}]'$ is called the adjustment vector.

From the equation, M_0 is called the key block (incomplete) of a single factorial effect confounded design. For a plan of simultaneous confounding of two factorial effects in a 5^n - factorial experiment, we are to follow the following steps.

Step 1. Find the independent key incomplete blocks for the factorial effects to be confounded simultaneously using Equation (1). Let these key incomplete blocks be denoted by, M_0, represents the level combinations of key incomplete block for the first confounded factorial effect and M_0' represents the level combinations of key incomplete block for the second confounded factorial effect.

Step 2. Find the common elements of level combinations (row vectors) of these two key incomplete blocks and form a matrix, can be denoted by B_1, called the key intrablock subgroup of level combinations of a two factorial effects confounded simultaneously in a 5^n factorial experiment. It can be seen that the key intrablock subgroup contains the lowest level combination for all the factors.

Step 3. Proceeding column wise we will get the second intrablock subgroup B_2 by adding the vector of level combination $(\ldots 0\ 1\ 0)$ with each of the elements of key intrablock subgroup. We get the third intrablock subgroup B_3 by adding the vector of level combination $(\ldots 0\ 2\ 0)$ with each of the elements of key intrablock, the fourth and fifth intrablocks B_4, B_5 can be obtained by adding the vectors of level combination $(\ldots 0\ 3\ 0)$ and $(\ldots 0\ 4\ 0)$ respectively with each of the elements (row vectors) of the key intrablock subgroup. We can place these intrablock subgroups in the first column of our final design and these are numbered as B_1, B_2, B_3, B_4 and B_5.

Proceeding row wise we will get in the second column the sixth, seventh, eighth, ninth and tenth intrablock subgroups $(B_6, B_7, B_8, B_9$ and $B_{10})$ by adding the vector $(\ldots 0\ 0\ 1)$ with each of the vector elements of B_1, B_2, B_3, B_4 and B_5, intrablock subgroups respectively in the first column of our final design.

To obtain the intrablock subgroups in the third column, B_{11}, B_{12}, B_{13}, B_{14} and B_{15}, we will add the vector $(\ldots 0\ 0\ 2)$ with each of the row vector elements of B_1, B_2, B_3, B_4 and B_5 respectively.

For the intrablock subgroups in the fourth column, B_{16}, B_{17}, B_{18}, B_{19} and B_{20}, we will add the vector $(\ldots 0\ 0\ 3)$ with each of the row vector elements of B_1, B_2, B_3, B_4 and B_5 respectively.

Similarly, for the intrablock subgroups in the fifth column, B_{21}, B_{22}, B_{23}, B_{24} and B_{25}, we will add the vector $(\ldots 0\ 0\ 4)$ with each of the row vector elements of B_1, B_2, B_3, B_4 and B_5 respectively.

In the final design, we get twenty five intrablock subgroups in five columns, each of which has five intrablock subgroups. A notational arrangement of the intrablocks of the final design is shown below.

Col.1	Col.2	Col.3	Col.4	Col.5
B_1	B_6	B_{11}	B_{16}	B_{21}
B_2	B_7	B_{12}	B_{17}	B_{22}
B_3	B_8	B_{13}	B_{18}	B_{23}
B_4	B_9	B_{14}	B_{19}	B_{24}
B_5	B_{10}	B_{15}	B_{20}	B_{25}

Example. Suppose we are to construct the layout of 5^3-factorial experiment where the factorial effects ABC and ABC^2 are confounded simultaneously.

The key incomplete block may be confounded with ABC is given by,

$M_0 = [V_1\{5^0\}, V_2\{5^1\}, a_u]_{5^{n-1}\times 3}$, with

$V_1\{5^0\} = 5^0[0I_5\ 1I_5\ 2I_5\ 3I_5\ 4I_5]'$;

$V_2\{5^1\} = 5[0I_1\ 1I_1\ 2I_1\ 3I_1\ 4I_1]'$ and the adjustment vector a_u can be obtained by solving the symbolic equation $x_1 + x_2 + x_3 = 0$ mod 5, taking first two values of x_1 and x_2 from the vectors V_1 and V_2 respectively for ABC and similarly, the key incomplete block confounded with ABC^2 is given by,

$M_0^{/} = [V_1\{5^0\}, V_2\{5^1\}, a_u]_{5^{n-1}\times 3}$, with

$V_1\{5^0\} = 5^0[0I_5\ 1I_5\ 2I_5\ 3I_5\ 4I_5]'$;

$V_2\{5^1\} = 5[0I_1\ 1I_1\ 2I_1\ 3I_1\ 4I_1]'$ and the adjustment vector a_u can be obtained by solving the symbolic equation $x_1 + x_2 + 2x_3 = 0$ mod 5, taking first two values of x_1 and x_2 from the vectors V_1 and V_2 respectively for ABC^2.

$$M_0 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 4 \\ 0 & 2 & 3 \\ 0 & 3 & 2 \\ 0 & 4 & 1 \\ 1 & 0 & 4 \\ 1 & 1 & 3 \\ 1 & 2 & 2 \\ 1 & 3 & 1 \\ 1 & 4 & 0 \\ 2 & 0 & 3 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \\ 2 & 3 & 0 \\ 2 & 4 & 4 \\ 3 & 0 & 2 \\ 3 & 1 & 1 \\ 3 & 2 & 0 \\ 3 & 3 & 4 \\ 3 & 4 & 3 \\ 4 & 0 & 1 \\ 4 & 1 & 0 \\ 4 & 2 & 4 \\ 4 & 3 & 3 \\ 4 & 4 & 2 \end{bmatrix} \qquad M_0' = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 2 & 4 \\ 0 & 3 & 1 \\ 0 & 4 & 3 \\ 1 & 0 & 2 \\ 1 & 1 & 4 \\ 1 & 2 & 1 \\ 1 & 3 & 3 \\ 1 & 4 & 0 \\ 2 & 0 & 4 \\ 2 & 1 & 1 \\ 2 & 2 & 3 \\ 2 & 3 & 0 \\ 2 & 4 & 2 \\ 3 & 0 & 1 \\ 3 & 1 & 3 \\ 3 & 2 & 0 \\ 3 & 3 & 2 \\ 3 & 4 & 4 \\ 4 & 0 & 3 \\ 4 & 1 & 0 \\ 4 & 2 & 2 \\ 4 & 3 & 4 \\ 4 & 4 & 1 \end{bmatrix}$$

Step 1.

Select the common elements (vectors) of M_0 and M_0' to find the key intrablock subgroup B_1, as shown below.

$$B_1 = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 4 & 0 \\ 2 & 3 & 0 \\ 3 & 2 & 0 \\ 4 & 1 & 0 \end{bmatrix}$$

Step 2.

We get the second intrablock subgroup B_2 by adding the vector $(0\ 1\ 0)$ to each of the vectors of the key intrablock subgroup; B_2 is placed just below (column side) the key block B_1.

Similarly, we get the third, fourth and fifth intrablocks B_3, B_4 and B_5 by adding the vectors $(0\ 2\ 0)$, $(0\ 3\ 0)$ and $(0\ 4\ 0)$ to each of the vectors of the key intrablock subgroups respectively, shown below.

Step 3.

After getting the intrablock subgroups, B_1, B_2, B_3, B_4 *and* B_5 we get the intrablock subgroups $B_6, B_7, B_8,$ B_9 and B_{10} by adding the vector $(0\ 0\ 1)$ to each of the vector elements (row vectors) of B_1, B_2, B_3, B_4 and B_5 respectively.

Similarly, an addition of the vector $(0\ 0\ 2)$ to each of the vector elements of B_1, B_2, B_3, B_4 and B_5 will produce the intrablock subgroups $B_{11}, B_{12}, B_{13}, B_{14}$ and B_{15}.

An addition of the vector $(0\ 0\ 3)$ to each of the vector elements of B_1, B_2, B_3, B_4 and B_5 will produce the intrablock subgroups B_{16}, B_{17}, B_{18}, B_{19} and B_{20}; and

An addition of the vector $(0\ 0\ 4)$ to each of the vector elements of B_1, B_2, B_3, B_4 and B_5 will produce the intrablock subgroups $B_{21}, B_{22}, B_{23}, B_{24}$ and B_{25}, which completes the plan, shown below.

Simultaneous confounding of ABC and ABC^2 in a 5^3 - factorial experiment, the generalized interactions those are also confounded could be found as:

$ABC \times ABC^2 = (A^2B^2C^3)^3 = ABC^4$;

$ABC \times (ABC^2)^2 = A^3B^3C^5 = AB$;

$ABC \times (ABC^2)^3 = A^4B^4C^7 = (A^4B^4C^7)^4 = ABC^3$ and

$ABC \times (ABC^2)^4 = A^5B^5C^9 = C$.

$$\begin{bmatrix}
 & 0 & 0 & 0 & & 0 & 0 & 1 & & 0 & 0 & 2 & & 0 & 0 & 3 & & 0 & 0 & 4 \\
 & 1 & 4 & 0 & & 1 & 4 & 1 & & 1 & 4 & 2 & & 1 & 4 & 3 & & 1 & 4 & 4 \\
B1 & 2 & 3 & 0 & B6 & 2 & 3 & 1 & B11 & 2 & 3 & 2 & B16 & 2 & 3 & 3 & B21 & 2 & 3 & 4 \\
 & 3 & 2 & 0 & & 3 & 2 & 1 & & 3 & 2 & 2 & & 3 & 2 & 3 & & 3 & 2 & 4 \\
 & 4 & 1 & 0 & & 4 & 1 & 1 & & 4 & 1 & 2 & & 4 & 1 & 3 & & 4 & 1 & 4 \\
 & 0 & 1 & 0 & & 0 & 1 & 1 & & 0 & 1 & 2 & & 0 & 1 & 3 & & 0 & 1 & 4 \\
 & 1 & 0 & 0 & & 1 & 0 & 1 & & 1 & 0 & 2 & & 1 & 0 & 3 & & 1 & 0 & 4 \\
B2 & 2 & 4 & 0 & B7 & 2 & 4 & 1 & B12 & 2 & 4 & 2 & B17 & 2 & 4 & 3 & B22 & 2 & 4 & 4 \\
 & 3 & 3 & 0 & & 3 & 3 & 1 & & 3 & 3 & 2 & & 3 & 3 & 3 & & 3 & 3 & 4 \\
 & 4 & 2 & 0 & & 4 & 2 & 1 & & 4 & 2 & 2 & & 4 & 2 & 3 & & 4 & 2 & 4 \\
 & 0 & 2 & 0 & & 0 & 2 & 1 & & 0 & 2 & 2 & & 0 & 2 & 3 & & 0 & 2 & 4 \\
 & 1 & 1 & 0 & & 1 & 1 & 1 & & 1 & 1 & 2 & & 1 & 1 & 3 & & 1 & 1 & 4 \\
B3 & 2 & 0 & 0 & B8 & 2 & 0 & 1 & B13 & 2 & 0 & 2 & B18 & 2 & 0 & 3 & B23 & 2 & 0 & 4 \\
 & 3 & 4 & 0 & & 3 & 4 & 1 & & 3 & 4 & 2 & & 3 & 4 & 3 & & 3 & 4 & 4 \\
 & 4 & 3 & 0 & & 4 & 3 & 1 & & 4 & 3 & 2 & & 4 & 3 & 3 & & 4 & 3 & 4 \\
 & 0 & 3 & 0 & & 0 & 3 & 1 & & 0 & 3 & 2 & & 0 & 3 & 3 & & 0 & 3 & 4 \\
 & 1 & 2 & 0 & & 1 & 2 & 1 & & 1 & 2 & 2 & & 1 & 2 & 3 & & 1 & 2 & 4 \\
B4 & 2 & 1 & 0 & B9 & 2 & 1 & 1 & B14 & 2 & 1 & 2 & B19 & 2 & 1 & 3 & B24 & 2 & 1 & 4 \\
 & 3 & 0 & 0 & & 3 & 0 & 1 & & 3 & 0 & 2 & & 3 & 0 & 3 & & 3 & 0 & 4 \\
 & 4 & 4 & 0 & & 4 & 4 & 1 & & 4 & 4 & 2 & & 4 & 4 & 3 & & 4 & 4 & 4 \\
 & 0 & 4 & 0 & & 0 & 4 & 1 & & 0 & 4 & 2 & & 0 & 4 & 3 & & 0 & 4 & 4 \\
 & 1 & 3 & 0 & & 1 & 3 & 1 & & 1 & 3 & 2 & & 1 & 3 & 3 & & 1 & 3 & 4 \\
B5 & 2 & 2 & 0 & B10 & 2 & 2 & 1 & B15 & 2 & 2 & 2 & B20 & 2 & 2 & 3 & B25 & 2 & 2 & 4 \\
 & 3 & 1 & 0 & & 3 & 1 & 1 & & 3 & 1 & 2 & & 3 & 1 & 3 & & 3 & 1 & 4 \\
 & 4 & 0 & 0 & & 4 & 0 & 1 & & 4 & 0 & 2 & & 4 & 0 & 3 & & 4 & 0 & 4
\end{bmatrix}$$

Considering the blocks in columns we have,

$(B_1 + B_2 + B_3 + B_4 + B_5)$ Vs.
$(B_6 + B_7 + B_8 + B_9 + B_{10})$ Vs.
$(B_{11} + B_{12} + B_{13} + B_{14} + B_{15})$ Vs.
$(B_{16} + B_{17} + B_{18} + B_{19} + B_{20})$ Vs.
$(B_{21} + B_{22} + B_{23} + B_{24} + B_{25})$; confounds the effect C, a generalized interaction of ABC and ABC^2.

Considering the blocks in rows we have,

$(B_1 + B_6 + B_{11} + B_{16} + B_{21})$ Vs.
$(B_2 + B_7 + B_{12} + B_{17} + B_{22})$ Vs.
$(B_3 + B_8 + B_{13} + B_{18} + B_{23})$ Vs.
$(B_4 + B_9 + B_{14} + B_{19} + B_{24})$ Vs.
$(B_5 + B_{10} + B_{15} + B_{20} + B_{25})$; confounds the effect AB, a generalized interaction of ABC and ABC^2.

Considering I - totals, we have,

$(B_1 + B_7 + B_{13} + B_{19} + B_{25})$ Vs.
$(B_2 + B_8 + B_{14} + B_{20} + B_{21})$ Vs.
$(B_3 + B_9 + B_{15} + B_{16} + B_{22})$ Vs.
$(B_4 + B_{10} + B_{11} + B_{17} + B_{23})$ Vs.
$(B_5 + B_6 + B_{12} + B_{18} + B_{24})$; confounds the effect ABC^4, a generalized interaction of ABC and ABC^2.

Considering J - totals, we have

$(B_1 + B_{10} + B_{14} + B_{18} + B_{22})$ Vs.
$(B_2 + B_6 + B_{15} + B_{19} + B_{23})$ Vs.
$(B_3 + B_7 + B_{11} + B_{20} + B_{24})$ Vs.
$(B_4 + B_8 + B_{12} + B_{16} + B_{25})$ Vs.
$(B_5 + B_9 + B_{13} + B_{17} + B_{21})$; confounds the effect ABC.

Considering I - totals and J-totals after rearranging the rows of the intrablocks in the final design at a two steps interval, we can find the effects ABC^2 and ABC^3 are confounded respectively.

A comparison of the blocks,

$(B_1 + B_9 + B_{12} + B_{20} + B_{23})$ Vs.
$(B_2 + B_{10} + B_{13} + B_{16} + B_{24})$ Vs.
$(B_3 + B_6 + B_{14} + B_{17} + B_{25})$ Vs.
$(B_4 + B_7 + B_{15} + B_{18} + B_{21})$ Vs.
$(B_5 + B_8 + B_{11} + B_{19} + B_{22})$; confounds the effect ABC^2.

A comparison of the blocks,

$(B_1 + B_8 + B_{15} + B_{17} + B_{24})$ Vs.
$(B_2 + B_9 + B_{11} + B_{18} + B_{25})$ Vs.
$(B_3 + B_{10} + B_{12} + B_{19} + B_{21})$ Vs.
$(B_4 + B_6 + B_{13} + B_{20} + B_{22})$ Vs.
$(B_5 + B_7 + B_{14} + B_{16} + B_{23})$; confounds the effect ABC^3.

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