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Hu Moment-based Reduced Feature Vector Analysis

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Abstract

For classification, features are extracted and converted into appropriate feature sets. In this paper, concentration is focused on to reduce the size of the feature vector that is constructed based on seven Hu invariants. The notion of reduced size of Hu moment is really interesting. Since its inception, seven higher-order Hu moments have been employed by many researchers without exploring 'why seven', and why not less numbers of moments. In this paper, we analyzed with various feature vector (FV) sets, which are composed of different combinations of Hu moments and rationalized that based on the characteristics of central moments, it is not necessary to employ all the seven moments in every applications. Through this manner, we can reduce the computational cost. Based on various FV sets, it is evident that we can use lower dimensional feature vectors for various methods of action recognition. Our various experimental evaluations provide evidence that up to the first two or three invariants are sufficient for history images and if we consider energy images, then only the 1st invariant for both images seem adequate for satisfactory recognition.

I. Introduction

Recognizing the identity of individuals as well as the actions, activities and behaviors performed by one or more persons in video sequences are very important for various applications. Surveillance, robotics, rehabilitation, video indexing, in the fields of biomechanics, medicine, sports analysis, film, games, mixed reality, etc. are among various key application arenas of human motion recognition[1-2]. For various pattern recognition methods, moment invariants are widely employed[1,7]. Moment invariants were firstly introduced to the pattern recognition community by M.K. Hu[15], who employed the results of the theory of algebraic invariants and derived his seven famous invariants to rotation of 2D objects. Since its inception, the Hu's invariants became classical. The use of moments for image analysis, object representation and recognition was inspired by these invariants. However, both for image reconstruction and pattern recognition, researchers employ all the seven invariants in their work for shape analysis. Even though it is assumed that higher order moments are less stable, detailed experimental analysis has been unattended and hence, all the invariants are computed. In general, gross image shape is represented well by the lower-order moments, and high-order moments only reflect the subtleties of a silhouette or boundary image. We find that complexity increases dramatically with increasing order and their containment of redundant information about shape.

In this paper, these seven invariants are computed to calculate feature vectors by employing the MHI[8] method, the HMHH[33] method and the Directional Motion History Image (DMHI) method[3-4]. For an action sequence, four history images (called, DMHI images) and four energy images (called, DMEI images) are produced. These eight motion templates are required for further motion representations. We employed seven moments to one moment, with and without normalized 0th order moments, to create different feature vector sets for classification. Higher the order for invariants, more noisier and unstable natures they demonstrate[7]. It seems that 0th order moment is not required to consider for history images. However, for energy images, the normalized 0th order moments demonstrate sufficient information on the motion-occurring region. Therefore, these feature vectors can significantly reduce the computational cost without sacrificing the recognition rates.

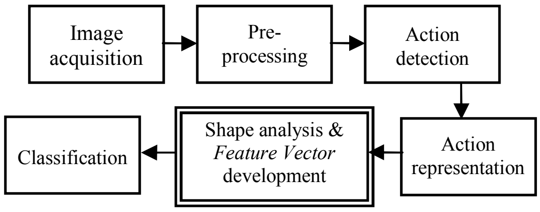

Fig. 1. System flow-diagram for a typical classification process.

Fig. 1 demonstrates a flow-diagram for approaches of this paper. For 'action representation', we consider the DMHI, the HMHH and the MHI method. For feature vector development, this paper concentrates on Hu invariants. However, there are numerous approaches to get the shape for calculating feature vectors from image templates. Fig. 2 gives an overview of various options for shape representations[1]. Though Hu invariants are widely employed for the MHI or related methods, other approaches, e.g., Zernike moments, global geometric shape descriptors, Fourier transform are also utilized for creating feature vectors[1].

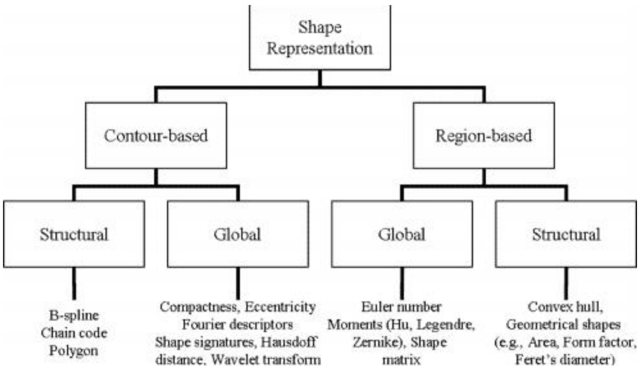

Fig. 2. Different approaches for computing feature vectors.

The paper is organized as follows: Section II covers some related works on moment-based recognition issues. Section III presents the moments. In Section IV, we demonstrate our concept of reduced feature vectors and analyzed the concept employing some recognition methods (the MHI, the HMHH and the DMHI methods). Finally, we conclude our paper in Section V.

II. Related Works

Hu moments are widely used by many researchers for image representation or image reconstruction, utilized as pattern recognition features in different applications (e.g., action recognition[1,3,8,11,32], classification[9,17] of textures, character recognition[12,22], image normalization[14], estimation of the position and the attitude of the object in 3-D space[16], for fingerprint verification[24], rapid matching of video streams, data matching[23]). We know that overall image shape is represented well by the lower-order moments, and high-order moments only reflect the subtleties of a silhouette or boundary image[5]. Therefore, one claim is that it may not be necessary to have all seven moment invariant functions to design a classifier[5]. The survey[18] on moment-based techniques note that most of the practical experiments have shown little improvement in identification performance when moment orders are increased beyond order four or five, and in general, high order moments are very sensitive to noise and they lack stability[7,10,18,20-21]. Also, higher order moments are more vulnerable to white noise[21].

First four invariants are exploited in one research[13]. For shape analysis and classification of weld defects in industrial radiography, Hu invariants are applied with other geometric parameters[17]. The first two invariants give measures in relation with the pixel spreading in comparison with the center of mass[17]. The first two moment invariants are used by Hu[15] to represent several known digitized patterns in a two-dimensional feature space. Similarly, Rizon et al.[19] use invariants for object detection and it was found from their works that from 2nd invariant onward, the invariants are less significant. It is possible to see that the 4th and the 5th moments have the larger standard deviations and that the moments are less stable under rotation and scaling[6]. This is due to the fact that higher order moments are the less stable. In order to maintain a good stability, a smaller set of moments has been selected, composed by the five moments in gray-scale image. The higher is the order of the moment, the higher are the fluctuations[6]. As all the complex moments are approximate, the last moments are the less stable, because they depend on higher power of uncertain numbers.

Apart from the above discussion, these invariants have information redundancy[10]. Also, the computed Hu invariant moments in the presence of noise, begin to degrade. Large variation in the dynamic range of values may create instability[10].

Apart from the concept of reducing the number of invariants, various dimensional reduction approaches[26-31] are employed, (e.g., Principal Component Analysis (PCA), Independent Components Analysis (ICA), Fisher Linear Discriminant Analysis (LDA), etc.) to reduce the dimensions of feature vectors. Dimension reduction is the process of mapping high-dimensional data into a low-dimensional space[26]. Since images are often represented as high-dimensional pixel arrays, suitable dimension reduction techniques need to be applied to increase efficiency. Subspace methods have been one of the most popular techniques for the dimension reduction. In Principal Component Analysis (PCA)[27], first calculate the linear subspace spanned by the principal directions with the largest variations, and to then project a sample vector into it. A nonlinear variant of PCA, called Kernel PCA (KPCA)[28] presumes that there exists a nonlinear map that makes mapped observations linearly separable, and then applies PCA to them. By contrast, a generalization of PCA known as independent components analysis (ICA)[29] learns a set of statistically independent components by analyzing the higher-order dependencies in the training data in addition to the correlations[30]. Multi-linear variants, called Multi-linear ICA (MICA) and Multi-linear PCA (MPCA) are employed to analyze the interaction between multiple factors using a tensor framework[30].

III. Lower-dimensional FV

We employ Hu moments to develop feature vectors for the DMHI/MHI/HMHH representations for each activity. To define the 2D $(p+q)^{th}$ order Cartesian moment m_{pq}, of a density distribution function $\rho(x,y)$, we can write,

$$m_{pq} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^p y^q \rho(x,y)\,dx\,dy \quad (p,q = 0,1,2,\ldots)$$

where, p and q are the order of the moments in the x and y axes, respectively. A complete moment set of order n consists of all moments, m_{pq}, such that $p+q \le n$ and contains $\frac{1}{2}(n+1)(n+2)$ elements. The moment transformation for a 2D digital image $f(x,y)$, of size (N,M) is defined by,

$$m_{pq} = \frac{1}{NM}\sum_{x=1}^{N}\sum_{y=1}^{M} x^p y^q f(x,y)$$

The next step is to make the moments invariant with respect to scale, position, and orientation. This is accomplished based on the theories of invariant algebra that deals with the properties of certain classes of algebraic expressions which remain invariant under general linear transformations[15]. We compute central moments in order to convert it to invariance under translation. Hence, the central moment μ_{pq} is calculated by,

$$\mu_{pq} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (x-\bar{x})^p (y-\bar{y})^q \rho(x,y) d(x-\bar{x}) d(y-\bar{y})$$

where, $\bar{x} = m_{10}/m_{00}$, $\bar{y} = m_{01}/m_{00}$. This is essentially a translated Cartesian moment, which means that the centralized moments are invariant under translation. The first four orders (i.e., $(p+q)$ is from 0 to 3) central moments are as follows,

$$\begin{aligned}
\mu_{00} &= m_{00} \equiv \mu, \\
\mu_{10} &= \mu_{01} = 0, \\
\mu_{20} &= m_{20} - \mu\bar{x}^2, \\
\mu_{11} &= m_{11} - \mu\bar{x}\,\bar{y}, \\
\mu_{02} &= m_{02} - \mu\bar{y}^2, \\
\mu_{30} &= m_{30} - 3m_{20}\bar{x} + 2\mu\bar{x}^3, \\
\mu_{21} &= m_{21} - m_{20}\bar{y} - 2m_{11}\bar{x} + 2\mu\bar{x}^2\bar{y}, \\
\mu_{12} &= m_{12} - m_{02}\bar{x} - 2m_{11}\bar{y} + 2\mu\bar{x}\,\bar{y}^2, \\
\mu_{03} &= m_{03} - 3m_{02}\bar{y} + 2\mu\bar{y}^3.
\end{aligned}$$

Based on the second and third order moments, we get six absolute orthogonal invariants,

$$\begin{aligned}
I_1 &= \mu_{20} + \mu_{02}, \\
I_2 &= (\mu_{20} - \mu_{02})^2 + 4\mu_{11}^2, \\
I_3 &= (\mu_{30} - 3\mu_{12})^2 + (3\mu_{21} - \mu_{03})^2, \\
I_4 &= (\mu_{30} + \mu_{12})^2 + (\mu_{21} + \mu_{03})^2, \\
I_5 &= (\mu_{30} - 3\mu_{12})(\mu_{30} + \mu_{12})[(\mu_{30} + \mu_{12})^2 - \\
&\quad 3(\mu_{21} + \mu_{03})^2] + (3\mu_{21} - \mu_{03})(\mu_{21} + \mu_{03})[3(\mu_{30} + \mu_{12})^2 \\
&\quad - (\mu_{21} + \mu_{03})^2], \\
I_6 &= (\mu_{20} - \mu_{02})[(\mu_{30} + \mu_{12})^2 - (\mu_{21} + \mu_{03})^3] + \\
&\quad 4\mu_{11}(\mu_{30} + \mu_{12})(\mu_{21} + \mu_{03}),
\end{aligned}$$

and one skew orthogonal invariant,

$$\begin{aligned}
I_7 = {} & (3\mu_{21} - \mu_{03})(\mu_{30} + \mu_{12})[(\mu_{30} + \mu_{12})^2 - 3(\mu_{21} + \mu_{03})^2] - \\
& (\mu_{30} - 3\mu_{12})(\mu_{21} + \mu_{03})[3(\mu_{30} + \mu_{12})^2 - (\mu_{21} + \mu_{03})^2].
\end{aligned}$$

Among these, the first six moments are invariants to rotation, scaling and translation. The 7th moment is skew and bi-correlations invariant that enables it to distinguish mirror or otherwise identical images. We employ these seven invariants to compute feature vector sets.

In this paper, we use the DMHI/MHI/HMHH methods to demonstrate the features of these invariants. For the DMHI method, to cover the overwriting issue significantly, normalized 0th order Hu moments ($\overline{m_{00}^{\varpi}}$ where, $\varpi \in \{up, down, left, right\}$. It defines four different directions of the DMHI method) are exploited[3-4]. The 0th order moments provide the total object area or mass, and hence provide important cues for motion region[25]. For the DMHI method, traditionally, seven invariants for each directional images, along with eight normalized 0th order moment invariants are calculated. Lets define the 0th order moment for a directional image ($H_\tau^{\varpi}(x,y,t)$) as,

$$m_{00}(H_\tau^{\varpi}) = \sum_x \sum_y H_\tau^{\varpi}(x,y,t)$$

The normalized normalized 0th order moment for a direction (e.g., for right direction of an action, $H_\tau^{+x}(x,y,t)$) can be computed by,

$$\overline{m_{00}^{+x}} = \frac{m_{00}\left(H_\tau^{+x}\right)}{m_{00}\left(H_\tau^{+x}\right) + m_{00}\left(H_\tau^{-x}\right) + m_{00}\left(H_\tau^{+y}\right) + m_{00}\left(H_\tau^{-y}\right)}$$

These normalized 0th order invariants confer the relative mass of the motion area in the scene. However, we notice that for Hu moment, complexity increases dramatically with increasing order.

IV. Experimental Results and Discussion

In our experiments, we employ six different action sets of traditional Japanese exercise (called *rajio-taiso*), taken from frontal-view video camera from various subjects. These exercises are complex in dimension and these are[25]: (i) Body stretching: raise both arms from the front and stretch, bring down the arms from the side, repeat these movements twice; (ii) Waving arms, then bend and straighten legs: bend and straighten the leg while waving arms, bend legs a bit while wave arms and straighten legs while bring down the arms; (iii) Turning the arms: turn both arms outward from bottom, and do it in reverse direction; (iv) Bending the chest: wave arms sideways while widening the gap between legs aside, and do the reverse; (v) Bending body sideways: raise the right arm from aside and bend the body to left, then be straight, do the reverse in right direction, back to normal posture; and finally (vi) Bending the body – front and back: bend upper part of the body at front, the posture will do three times as if to touch the floor, then straighten up and put a hand on the waist and bend the upper part of the body behind and take deep breath, and finally straighten to normal posture.

For classification, k-nearest neighbor classifier is employed, and leave-one-out cross-validation partitioning scheme is considered for recognition. Table 1 presents twelve different feature vector sets employing Hu moments for the DMHI templates[25]. We employed seven moments to one moment, with and without normalized 0^{th} order moments, to create 12 different feature vectors for classification. We also considered moments only for history templates and only for energy templates and achieved different recognition results. It seems that 0^{th} order moment is not required to consider, though for energy images, it can give key information about the mass of the motion area and hence, we can ignore to employ seven moments for energy images. However, instead of using seven moments for energy images, we can consider only the 0^{th} order moments that provide the total object area, as it seems sufficient.

Table 1. Recognition rate with different FV dimensions (FV_D) using DMHI method ($\aleph$ means no. of changes in the recognition)

FV_D	Hu moments		Rec. rate (%)
	DMHI	***DMEI***	
FV_{64D}	$7+\overline{m_{00}^{\varpi}}$	$7+\overline{m_{00}^{\varpi}}$	94.4
FV_{56D}	7	7	89.0
FV_{48D}	6	6	89.0
FV_{40D}	5	5	89.0
FV_{36D}	$7+\overline{m_{00}^{\varpi}}$	$\overline{m_{00}^{\varpi}}$	97.3
$FV_{32D0^{th}}$	7	$\overline{m_{00}^{\varpi}}$	89.0
FV_{32D}	4	4	89.0
FV_{24D} **$\aleph$ =1**	3	3	89.0
FV_{16D} **$\aleph$ =5**	2	2	89.0
FV_{8D} **$\aleph$ =26**	1	1	89.0
FV_{4D} **$\aleph$ =63**	1	0	80.6
FV_{4eD} **$\aleph$ =many**	0	1	44.5

As evident from Table 1, considering all moments up to four moments to calculate feature vectors for the DMHI method seem sufficient, because satisfactory recognition rates are achieved throughout. Considering k=5 in the k-nearest neighbor method, we checked top five results for all recognitions and no change in its distribution has been found. Moreover, the computed Euclidean distances among these are almost the same even if we consider all or four moments. After that, we notice changes among the results even though the recognition result is not varying significantly. The various feature vector sets shown in Table 1 can be organized as follows,

$$FV_{64D} = (\forall h)(I_{1\to7}) \cup (\forall e)(I_{1\to7}) \cup (\forall h, \forall \varpi)\left(\overline{m_{00}^{\varpi}}\right) \cup (\forall e, \forall \varpi)\left(\overline{m_{00}^{\varpi}}\right).$$
$$FV_{56D} = (\forall h)(I_{1\to7}) \cup (\forall e)(I_{1\to7}).$$
$$FV_{40D} = (\forall h)(I_{1\to5}) \cup (\forall e)(I_{1\to5}).$$
$$FV_{36D} = (\forall h)(I_{1\to7}) \cup (\forall h)\left(\overline{m_{00}^{\varpi}}\right) \cup (\forall e)\left(\overline{m_{00}^{\varpi}}\right).$$
$$FV_{32D0^{th}} = (\forall h)(I_{1\to7}) \cup (\forall e)\left(\overline{m_{00}^{\varpi}}\right).$$
$$FV_{24D} = (\forall h)(I_{1\to3}) \cup (\forall e)(I_{1\to3}).$$
$$\ldots\ldots\ldots$$
$$FV_{8D} = (\forall h)(I_1) \cup (\forall e)(I_1).$$
$$FV_{4D} = (\forall h)(I_1).$$
$$FV_{4eD} = (\forall e)(I_1).$$

In these equations, h represents motion *h*istory image, and e represents motion *e*nergy images. The FV_{64D} considers all moments as well as normalized 0^{th} order moments in the creation of feature vector. On the other hand, the FV_{56D} consists of seven invariants for both history and energy templates; whereas, FV_{32D} set is composed with seven invariants for history images along with the normalized 0^{th} order moments of the energy images. Therefore, these feature vectors can significantly reduce the computational cost without sacrificing the recognition rates.

Similarly, Table 2 shows another experimental argument on the importance to deploy lower order moments by considering the Hierarchical Motion History Histogram (HMHH) method[33]. In this approach, they define some patterns P_i in the motion mask ($D(x,y,:)$) sequences, based on the number of connected '1', e.g.,

$$P_1 = 010, P_2 = 0110, P_3 = 01110, \cdots, P_M = \underbrace{01\cdots10}_{M\ 1s}$$

Then a subsequence $C_i = b_{n1}, b_{n2}, \ldots, b_{ni}$ is defined to denote the set of all sub-sequences of $D(x,y,:)$ as $\Omega\{D(x,y,:)\}$. Then for each pixel, the number of occurrences of each specific pattern P_i in the sequence of $D(x,y,:)$ is computed as shown,

$$HMHH(x,y,P_i) = \sum_j \mathbf{1}_{\{C_j = P_i | C_j \in \Omega\{D(x,y,:)\}\}}$$

Here, $\mathbf{1}$ is the indicator function. From each pattern P_i, one gray-scale image (called Motion History Histogram (MHH)) is constructed, and in aggregation of all MHH images, the Hierarchical MHH (HMHH) is produced.

Table 2. Recognition rate with different FVs using HMHH method

FV_D	**Description**	**Rec. rate (%)**
FV_{28D}	7 invariants	66.7
FV_{24D}	6 invariants	66.7
FV_{20D}	5 invariants	72.2
FV_{16D}	4 invariants	66.7
FV_{12D}	3 invariants	66.7
FV_{8D}	2 invariants	55.6

In this case, various feature vectors for four different patterns are considered. We notice that until the first three invariants, the recognition rate is almost unchangeable; however, if we employ less than the first three invariants for recognition, the recognition rate starts to decrease from the earlier rate. In this case, the recognition rate for HMHH method is low.

We also conduct experiment with the basic Motion History Image (MHI) method[1,8]. The MHI $H_\tau(x,y,t)$ can be computed from an update function $\Psi(x,y,t)$:

$$H_\tau(x,y,t) = \begin{cases} \tau & \text{if } \Psi(x,y,t)=1 \\ \max(0, H_\tau(x,y,t-1)-\delta) & \text{otherwise} \end{cases}$$

Here, (x,y) and t show the position and time, $\Psi(x,y,t)$ signals object's presence (or motion) in the current video image, the duration τ decides the temporal extent of the movement (e.g., in terms of frames), and δ is the decay parameter. This update function is called for every new video frame analyzed in the sequence. The Motion Energy Image (MEI) can be deduced from the MHI (by thresholding the MHI above zero),

$$E_\tau(x,y,t) = \begin{cases} 1 & \text{if } H_\tau(x,y,t) \geq 1 \\ 0 & \text{otherwise} \end{cases}$$

Table 3. Recognition rate with different FVs using MHI method

FV_D	$H_\tau(\,)$	$E_\tau(\,)$	**Rec. rate (%)**
FV_{14D}	7	7	68.6
FV_{12D}	6	6	68.6
FV_{10D}	5	5	68.6
FV_{8D}	4	4	68.6
FV_{6D}	3	3	68.6
FV_{4D}	2	2	68.6
FV_{2D}	1	1	68.6
FV_{4h1eD}	4	1	68.6
FV_{1h4eD}	1	4	68.6
FV_{7eD}	0	7	62.9
FV_{7hD}	7	0	37.1
FV_{4eD}	0	4	62.9
FV_{4hD}	4	0	37.1

Table 3 shows various feature vector sets for recognizing a few actions. In this Table, feature vector combinations from FV_{14D} until FV_{2D} combine seven or less Hu invariants for history and energy images. For example, FV_{4D} means first 2 moments for each history and energy templates. From the experiments, we come across a point that for the MHI method, energy images dominate the recognition rates. However, only history images or only energy images are not sufficient for image information. The 1st and 2nd invariants seem reasonable for recognition.

V. Conclusions

In this paper, moment-based reduced feature vector sets are proposed for action recognition by employing the MHI, the HMHH and the DMHI method. The Hu invariants are widely due to its simplicity to compute and less numbers (only seven at most, whereas Zernike moments or other cases, the number of moments are higher). Almost in every case, vision community employs all the seven invariants in their works – both in image reconstruction and pattern recognition and shape analysis. This paper presents Hu moment-based reduced feature vector sets for action recognition perspective. Various feature sets with different number of Hu moments are exploited using the MHI, the HMHH and the DMHI methods to explore the applicability of the invariants. It is found from these experimental analyses that lower dimensional feature vectors can be used for developing feature vector sets by using Hu moment. We hope that this concept of reduced feature vectors may be useful for other applications where Hu moments (or, other moments) are explored.

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