Short Communication

Dhaka Univ. J. Sci. 59(2): 287-288, 2011 (July)

Tangent Line Approximations to Represent the Patterns of Graphs of Some Functions

Khadija Tabassum, Shapla Shirin and Sarker Md. Sohel Rana

Department of Mathematics, Dhaka University, Dhaka-1000, Bangladesh

Received on 26. 08. 2010. Accepted for Publication on 20.02.2011

I. Introduction

Let us assume some special functions of the form $(x+k)^p - h$ or $a\,(x+k)^r + b\,(x-g)^q - h$, where p, q are rational numbers and a, b, k, g, h represent constants. An equation to represent all possible equations of tangent lines targeting of approximations to the graphs of the functions stated above have been introduced. CAS [MATHEMATICA] is used to generate the graphs.

Let $y = f(x)$ be a real-valued function. If P $(a, f(a))$and Q$(b, f(b))$ are two points on the curve of f, then the secant line joining P and Q will be a tangent line PT at the point P, when Q approaches P and produces the following slope m_{tan} of tangent line PT, that is,

$m_{tan} = \lim_{b \to a} \frac{f(b)-f(a)}{b-a}$. However, the equation of the said tangent line at $x = a$ will stand as (see Fig.1), [1, 3, 4],

$$y_{tan} = m_{tan}\,(x - a) + f(a) \qquad (1)$$

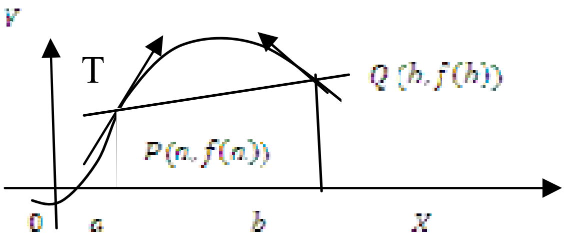

Fig.1. Tangent line.

Now, by plotting the tangent lines at each of the points in the interval $[a, b]$, one can visualize the tentative characteristic of the curve of a function which is termed as the tangent line approximations to the curve of the function at each points in the interval [a, b] (see Fig.2), [1, 3, 4]. We also find the region approximately where the curve is increasing, decreasing, concave up and concave down by these tangent line approximations. It is known to us that, in a region the curve of a function is increasing where the tangent has positive slope, decreasing where the tangent has negative slope, and concave up where the tangent lines are below the curve, concave down where the tangent lines are above the curve.

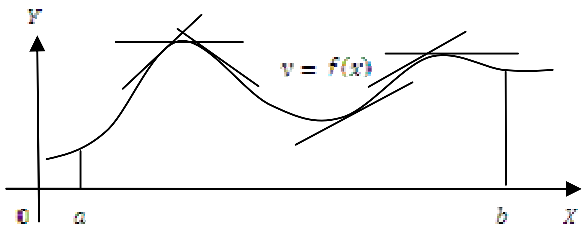

Fig. 2. Tangent line approximations.

II. Results and Discussion

Although, the family of tangent lines, [1 – 4] , to the function of the form

$$y = f(x) = (x+k)^p - h,$$

where p is a rational number and k, h are real constants can be obtained directly from the formula (1), we introduced here a different approach to get it from a known family of tangent lines to a known function by using transformations of axes.

Suppose that $y = f(x) = x^p$, p is rational number (2)

Then, $f(a) = a^p$ and the derivative of f at $x = a$ is $f'(a) = p\,a^{p-1}$. So, the equation of tangent line to the curve (2) at $x = a$ is $T(x,a) = pa^{p-1}x - (p-1)a^p$ (3)

If we shift the origin to the point $(k, 0)$, $k > 0$, then for the coordinates (x', y') according to the new axes, we have

$$x' = x - k \Rightarrow x = x' + k;$$

$$y' = y - 0 \Rightarrow y = y' + 0;$$

and $a' = a - k \Rightarrow a = a' + k.$

Now, according to the new axes, the equation (2) and (3) can be written as, respectively,

$$y = (x' + k)^p \qquad (4)$$

and

$$T(x,a) = p(a'+k)^{p-1}(x'+k) - (p-1)(a'+k)^p \qquad (5)$$

Again we shift the new origin at (k, h), $h > 0$, then the equations (4) and (5) will be of the form

$$y = (x' + k)^p - h \qquad (6)$$

and

$$T(x,a) = p(a'+k)^{p-1}(x'+k) - (p-1)(a'+k)^p - h \qquad (7)$$

Replacing x' by x and a' by a in (6) and (7), we get

$$y = (x+k)^p - h \qquad (8)$$

and

$$T(x,a) = p(a+k)^{p-1}(x+k) - (p-1)(a+k)^p - h \qquad (9)$$

Hence, equation (9) represents the family of tangent lines to the graph of (8) at $x = a$, a being a parameter, and p, k, h are real constants.

Considering different set of values of parameters in equations (8) and (9), the corresponding graphs are shown in the following figures (Fig.3 – Fig.6) where the first graphs of each are presented by the tangent line approximations approach and the second graphs are obtained directly.

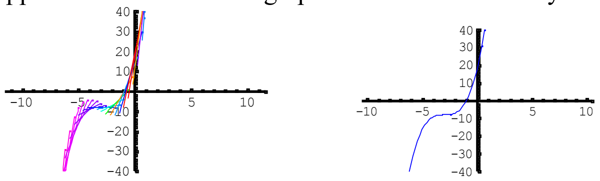

Fig. 3. p is a positive integer. $p = 3$, $k = 3$ and $h = 8$.

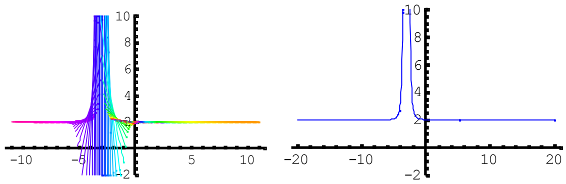

Fig. 4. p is a negative integer. $p = -4$, $k = 3$ and $h = -2$..

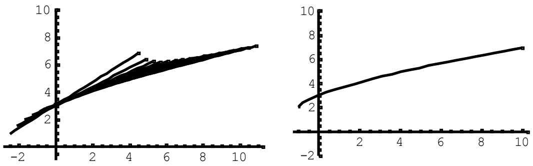

Fig. 5. p is a positive rational number. $p = {}^{2}/_{3}$, $k = 1$ and $h = -2$.

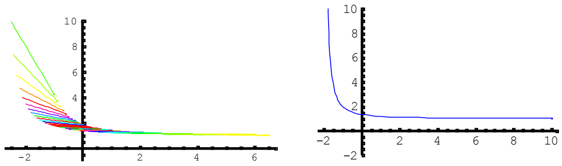

Fig.6. p is a negative rational number. $p = -{}^{2}/_{3}$, $k = 2$ and $h = -1$.

Remark (Generalization of equations (8) – (9)) : According to the equations (8) and (9), the equation

$$T(x, a) = \sum_{i=1}^{n} c_i \left[p_i (a + k_i)^{p_i - 1}(x + k_i) - (p_i - 1(a + k_i)^{p_i})\right] - h \tag{10}$$

represents the general form of the equations of all tangent lines at $x = a$, where a is a parameter, to the graph of the function

$$y = f(x) = \sum_{i=1}^{n} [c_i (x + k_i)^{p_i}] - h \tag{11}$$

As before, the following graphs are obtained from (10) - (11) :

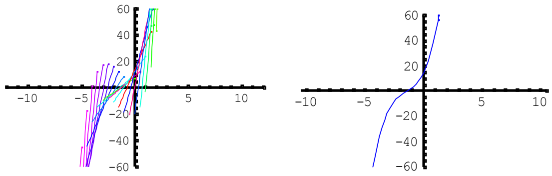

Fig. 7. $n = 3$, $p_1 = 3$, $p_2 = 2$, $p_3 = 1$, $c_1 = 2$, $c_2 = 3$, $c_3 = 1$, $k_1 = k_2 = 1, k_3 = 0$ and $h = 0$.

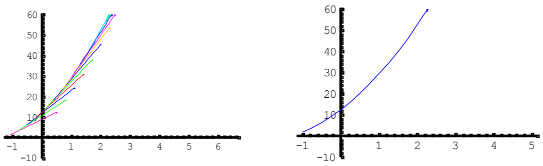

Fig. 8. $n = 3$, $p_1 = \frac{3}{2}$, $p_2 = 2$, $p_3 = 1$, $c_1 = \frac{1}{2}$, $c_2 = 3$, $c_3 = 1$, $k_1 = k_2 = 1, k_3 = 0$ and $h = 0$.

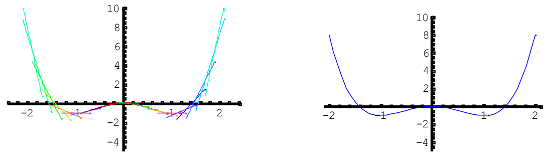

Fig. 9. $n = 3, p_1 = 4$, $p_2 = 2$, $c_1 = 1$, $c_2 = 2$, $k_1 = k_2 = 1, k_3 = 0$ and $h = 0$.

III. Conclusion

In literature [1 – 4], first and second derivative of a function were used for rough sketch of the function. In this paper, we found that the family of tangent lines of functions of the form $(x + k)^p - h$ or $a\,(x + k)^p + b\,(x + g)^q - h$, which is a good approximation to the graph of that function. We also found the pattern of the graph of a function that is quite similar to the actual graph of the function which we get from the other method. We can now say that, through this approach, one can easily visualize the pattern of the graph of a function.

[Remarks : The ideas which have been discussed in this article are not contained in literature [1 – 4]. So far of our knowledge, no research articles have been published related to this notion as yet.]

....

1. Anton H, *Calculus with Analytic Geometry*, Fifth Edition, John Wiley & Sons, INC, NewYork, United States of America, 1995.
2. Anton H, *Calculus with Analytic Geometry*, Fifth Edition, John Wiley & Sons, INC, NewYork, United States of America, 1995.
3. Bittinger M. L, B. B. Morrel *Applied Calculus*, Second Edition, Addison-Wesley Publishing Company, Inc., USA, 1988.
4. Swokowski E. W., M. Olinick and D. Pence *Calculus*, Sixth Edition, PWS Publishing Company, Boston, 1994.
5. Thomas G. B. and R. L. Finney *Calculus and Analytic Geometry*, Ninth Edition, Addison Wesley Longman,Inc. [Thomson Pres Ltd.,India],1998.