*Short communication*

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# Inter-Industry Linkage Indices: Methodological Development and Comparisons

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## I. Introduction

For quantitative national economic analysis, the inter-industry relation approach is a useful dimension of analysis and research. The nature and extent of inter-industry linkages is an issue of great importance for an economy of developing countries. The lack of linkage is of course one of the most typical characteristics of under developed economies. The linkage coefficient helps us to identify the key sectors of the economy in the strategy of country's development. It also helps us to identify the economic sectors due to their important position in the inter-industry network, which is significant for initiating or distributing growth impulses. A comprehensive study using the recent theoretical developments of inter-industry linkage indices is very useful tools to analyse the readily available input-output tables. Theoretical frameworks of inter-industry linkage indices are developed by some famous scientists since 1950 have been compiled and established their interrelations in this study.

## II. Development of Approaches to Inter-Industry Linkage Indices

Hirschman[1] first noted the idea of linkages as a means of identifying key sectors in development planning. He introduced the concept of 'backward' and 'forward' linkages and suggested methods of measuring them for industrial activity. Rasmussen[2] used the inverse of an input-output matrix in order to measure direct and indirect repercussions of an increase in the final requirements from any sector on other sector of the economy and he also developed measures of variability. Cella[3] proposed a more satisfactory technique, which defines the total linkage effect of an industry appropriately and then, identifies its two components backward and forward linkages. Meller and Morfan[4] computed employment linkages to identify key sectors instead of production linkages. Harrigan and McGilvray[5] proposed a hypothetical addition method for calculation of the internal and closed-loop linkages in addition to the linkage introduced by Cella. More recently, linkage analysis methods have again attracted increasing attention from the part of input-output analysis by Clements[6], Heimler[7], Sonis *et al.*[8] and Dietzenbacher and Vander Linden[9]. The most important methods of inter-industry linkage indices are as follows: Chenery-Watanabe, Rasmussen, Cella, Harrigan-McGilvray, Pure linkage and Dietzenbacher & Vander Linden Approaches.

## III. Comparisons among Different Linkage Approaches

### Relationship between Chenery-Watanabe and Rasmussen Methods

Rasmussen method of calculating backward and forward linkage is more improvement and more refinement compared to the backward and forward linkage measured by Chenery and Watanabe[10]. We can compare the backward linkage measures using two methods as follows: (Under usual notations)

$$\mathbf{BL}^{C} = \frac{\sum_i \mathbf{X}_{ij}}{\mathbf{X}_j} \quad \text{and} \quad \mathbf{BL}^{R} = \frac{\frac{1}{\mathbf{n}}\sum_i \mathbf{Z}_{ij}}{\frac{1}{\mathbf{n}^2}\sum_i\sum_j \mathbf{Z}_{ij}}$$

Similarly, $\mathbf{FL}^{C} = \frac{\sum_j \mathbf{X}_{ij}}{\mathbf{X}_i}$ and $\mathbf{FL}^{R} = \frac{\frac{1}{\mathbf{n}}\sum_j \mathbf{Z}_{ij}}{\frac{1}{\mathbf{n}^2}\sum_j\sum_i \mathbf{Z}_{ij}}$

### Relationship between Cella and Harrigan-McGilvray Methods

Backward linkages of Cella and Harrigan-McGilvray methods are

$$BL^{C} = i'[H - B_{11} + B_{22}A_{21}H]F_1 \text{ , Where,}$$
$$H = [I - A_{11} - A_{12}B_{22}A_{21}]^{-1}$$

And $BL^{G} = i'B_{22}A_{21}F_1$ , Where,

$$H = B_{11}(I - A_{12}B_{22}A_{21}B_{11})^{-1}$$

We know that $H=[I-A_{11}-A_{12}B_{22}A_{21}]^{-1} = B_{11}(1 - A_{12}B_{22}A_{21}B_{11})^{-1}$

Thus the value of H for both the equations is same

$$\text{Now, } BL^{C} = i'[H - B_{11} + B_{22}A_{21}H]F_1 =$$
$$i'(H - B_{11})F_1 + i'(B_{22}A_{21}H)F_1 =$$
$$i'(H - B_{11})F_1 + H(BL^{G})$$

Therefore, $BL^{C} \geq BL^{G}$ , Since, $H \geq 1$ and $i(H - B_{11})F_1 \geq 0$ (1)

Forward linkages of Cella and Harrigan-McGilvray methods are

$$FL^{C} = i'[HA_{12}B_{22} + B_{22}A_{21}HA_{12}B_{22}]F_2 \text{ and}$$
$$FL^{G} = i'A_{12}B_{22}F_2 ,$$

respectively.

Now, $FL^C = i'[HA_{12}B_{22} + B_{22}A_{21}HA_{12}B_{22}]F_2$ = $i'[(1 + B_{22}A_{21})HA_{12}B_{22}]F_2$ = $(1 + B_{22}A_{21})H[i'A_{12}B_{22}F_2]$ = $(H + B_{22}A_{21}H).(FL^G)$

Therefore, $FL^C \geq FL^G$, Since, $H \geq 1$ and $I + B_{22}A_{21} > 0$ (2)

**Relationship between Pure Linkage and Dietzenbacher - Vander Linden Linkage**

Backward linkages of the two methods are

$BL^P = e_2'(B_{22}A_{21}H)F_1 + e_2'(B_{22}A_{21}HA_{12}B_{22})F_2$, and

$BL^D = [(H - I) + e_2'B_{22}A_{21}H]F_1 / x_1 + [(H - I)A_{12}B_{22} + e_2'B_{22}A_{21}HA_{12}B_{22}]F_2 / x_1$

Respectively

Now, $BL^D = [(H - I) + e_2'B_{22}A_{21}H]F_1 / x_1 + [(H - I)A_{12}B_{22} + e_2'B_{22}A_{21}HA_{12}B_{22}]F_2 / x_1$

$= 1/x_1 [(H - I)F_1 + (H - I)A_{12}B_{22}F_2 + e_2'(B_{22}A_{21}H)F_1 + e_2'(B_{22}A_{21}HA_{12}B_{22})F_2]$

(Since $x_1 = HF_1 + HA_{12}B_{22}F_2$)

$$= [1 - \frac{1}{H} + \frac{BL^P}{HF_1 + HA_{12}B_{22}F_2}] \quad (3)$$

Forward linkages of the two methods are

$FL^P = e_2'(\hat{H}B_{12}Z_{22})F_1 + e_2'(Z_{22}B_{21}\hat{H}B_{12}Z_{22})F_2$, and

$FL^D = \frac{100}{x_2}[\{(\hat{H} - I)F_1 + \hat{H}B_{12}Z_{22}e_2'\}F_1 + e_2'\{Z_{22}B_{22}(\hat{H} - I) + Z_{22}B_{21}\hat{H}B_{12}Z_{22}\}F_2]$

Respectively, Where,

$\hat{H} = (I - B_{11} - B_{12}Z_{22}B_{21})^{-1}$ and $Z_{22} = (I - B_{22})^{-1}$

Now,

$FL^D = \frac{100}{x_2}[\{(\hat{H} - I)F_1 + \hat{H}B_{12}Z_{22}e_2'\}F_1 + e_2'\{Z_{22}B_{22}(\hat{H} - I) + Z_{22}B_{21}\hat{H}B_{12}Z_{22}\}F_2]$

$$= \frac{100}{x_2}[(\hat{H} - I)F_1 + e_2'\{Z_{22}B_{22}(\hat{H} - I)\}F_2 + FL^P]$$

$$= \frac{100}{(\hat{H} - I)F_1 + e_2'\{Z_{22}B_{22}(\hat{H} - I)\}F_2}[(\hat{H} - I)F_1 + e_2'\{Z_{22}B_{22}(\hat{H} - I)\}F_2 + FL^P]$$

$$= 100[1 + \frac{FL^P}{(\hat{H} - I)F_1 + e_2'\{Z_{22}B_{22}(\hat{H} - I)\}F_2}] \quad (4)$$

**Relationship between Cella, Harrigan-McGilvray and Pure Linkages**

The Pure backward linkage,

$BL^P =$

$e_2'(B_{22}A_{21}H)F_1 + e_2'(B_{22}A_{21}HA_{12}B_{22})F_2$

= [illegible]

$= BL^C - e_1'(H - B_{11})F_1 + e_2'(B_{22}A_{21}HA_{12}B_{22})F_2$ (5)

(Since $BL^C = i'[H - B_{11} + B_{22}A_{21}H]F_1$)

Where, $e_1'$ and $e_2'$ are the row summation vector for sector 1 and 2 respectively.

Again,

$BL^P =$

$e_2'(B_{22}A_{21}H)F_1 + e_2'(B_{22}A_{21}HA_{12}B_{22})F_2$

$= H.BL^G + e_2'(B_{22}A_{21}HA_{12}B_{22})F_2$ (6)

(Since $BL^G = i'B_{22}A_{21}F_1$)

**IV. Discussion**

The theoretical development to date in the linkage literature enables us to single out two main approaches: traditional and second generation ones. The traditional approach, as advanced by Rasmussen and the Cella linkage is named as "second generation linkage" which is modification of the Rasmussen model. The Rasmussen generates linkage index which is relative measures based on the size of the coefficients of the Leontief inverse, while the second generation linkages are absolute measures based on total sectoral output[11]. Hirschman[1] and Rostow[12] suggested that the development programme should initially be concentrated on certain key sectors via its technological ties with them. With respect to backward linkages, key sectors generate above-average inputs requirements from other sectors and are more likely to induce investment in the supply sector either by expanding the existing plans or replacing imports. With respect to forward linkages, the mechanism is less direct but continues to hinge on major inputs. Forward linkage is generally weaker since the output produced is not necessarily met by adequate demand. High forward linkage coefficients will be found in those sectors, which produce relatively little directly for final demand but rather for intermediate demand of other sectors. The backward linkage is more meaningful than forward linkage. Hirschman (1958) suggested that ideally we should start by investing in those industries, which are capable of generating induced investment in both backward and forward directions. If no such industry exists we should invest in industries with strong backward linkages because the pressures of excess

demand are altogether a more powerful, reliable and calculable method of stimulating economic development.

The linkage measures are used to indicate the role of linkages in industrial development and can be used as summary measures of structural interdependence of the economy. The Rasmussen index are used to see how the internal structure of the economy behaved, without taking into consideration the level of production in each sector and identifies what may be referred to as potential impacts from changes in any sector. On the other hand, the Cella, Harrigan-McGilvray and Pure linkage indices are used to consideration of the volume of activity. The difference between the Cella index and Harrigan-McGilvray index is minor and this is equal to the internal linkage. The characteristic of Cella's method is that it excludes purely internal transaction and it is based on a consistent input-output model of the economy with a fixed set of technical coefficients. Since Cella's method is based on the decomposition of the total output of all sectors, the backward and forward linkages obtained are not symmetrical, and they are not comparable to the corresponding linkage indicators given by Chenery-Watanabe and Rasmussen. The values of the pure linkage indices depend upon the size of final demand and hence these indices reflected the effects of a sector on the output of other sectors when final demand increases by one unit. For the pure linkage indices, the backward and forward linkage indices are summed to yield the total linkage indices and sectors, which have the greatest value of total linkage, are considered as key economic sectors[8].

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