Dhaka Univ. J. Sci. 58(2): 165-168, 2010 (July)

Matrix Method of Designing Simultaneous Confounding of Two Factorial Effects In 3^n - Factorial Experiment

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Received on 15. 03. 2009. Accepted for publication on 28. 10. 2009.

Abstract

This article describes matrix method of designing simultaneous confounding of two factorial effects in a 3^n - factorial experiment. It becomes easier to construct the design of simultaneous confounding for a 3^n - factorial experiment especially when the number of factors as well as the number of levels becomes larger.

I. Introduction

The practical works with factorial experiment become troublesome especially when the number of factors as well as the number of levels of each of the factors is large. This trouble becomes difficult if we have no required number of homogeneous plots in practice. In such situation, we are bound to use a limited number of homogeneous plots to analyze the factorial effects. As a result, some factorial effects or interactions will be mixed up with block effect, i.e. confounded. Since there is no way to avoid this, the higher order interaction effects are considered to be confounded. We usually consider one or two higher order interaction effects to be confounded to perform analysis efficiently.

Bose and Kishan (1940), Bose (1947) described the construction of p^n factorial designs using finite geometries. The treatments are represented by n-tuples $(a_1, \cdots\cdots, a_n)$ where a_i are elements of $GF(p)$. The method is available only when p is prime or prime power.

A system of simultaneous confounding in 2^n factorial experiment has been described, where an intrablock subgroup is constructed with the common elements taken from the factorial effects of two incomplete blocks, each confounded with a single factorial effect (Kempthorne, 1947, 1952)[1,2]. Das (1964) described an equivalent method of Bose in which some of the treatment factors are designated as basic factors and the others as added factors. Levels of added factors are derived by combination of the levels of the basic factors over $GF(p)$. White and Hultquist (1965) extended the field method to designs with numbers of levels of treatment factors. John and Dean (1975) described the construction of a particular class of single replicate block designs, which they call generalized cyclic designs. The essential feature of the method is that the n-tuples giving the treatments of a particular block constitute an Abelian group, the intrablock subgroup. Patterson (1976) described a general computer algorithm, called DSIGN, in which levels of treatment factors are derived by linear combinations of levels of plot and block factors. The method provides finite-field, generalized cyclic and other designs. Mallick, S. A. (1973 & 1975)[3, 4] developed two systems of designing factorial effects with simultaneous confounding of two effects, one for a 3^n and the other for a 4^n - factorial experiments. In all these systems of simultaneous confounding, the design of factorial effects was based on some manipulating manner, selection by inspection the common elements (factorial effects). Jalil, M. A. et al (1990) developed matrix method[5] of designing a single factorial effect confounded in a P^n - factorial experiment, where the level combinations are obtained by matrix operations of the levels. The present work is also a matrix method of designing a 3^n - factorial experiment of simultaneous confounding of two factorial effects.

II. Method

In the construction of a P^n - factorial experiment with a single factorial confounded, we can write the level combinations by the equation described below (Jalil, M. A. et al 1990).

$$M = [M_0, M_1, M_2, \cdots, M_{p-1}]; \qquad (1)$$

where incomplete blocks $M_u; u = 0,1,2,\cdots,(p-1)$ is given by:

$M_u = [V_1\{p^0\}, V_2\{p^1\}, \cdots, V_n\{p^{p-1}\}, a_u]_{p^{n-1}\times n}$ with

$V_i\{p^j\} = p^j[0 I_{p^{(n-1)-i}}, 0 I_{p^{(n-1)-i}}, \cdots, (p-1) I_{p^{(n-1)-i}}]$,

each is a column vector of dimension p^n.

$u = 0,1,2,\cdots,(p-1)$; $i = 1,2,3,\cdots,(n-1)$;
$j = 0,1,2,\cdots,(p-1)$; with the restriction that $i = j+1$;

I_m : sum vector of dimension m;

$\{p^j\} = p^j -$ times repetition of the elements of V_i in ascending ordered levels; and

$a_u = [a_{u1}, a_{u2}, \cdots, a_{up}]'$ is called the adjustment vector.

From the equation matrices (Eq. 1), M_0 is called the key block (incomplete) of a single factorial effect confounded design. For a plan of simultaneous confounding of two factorial effects in a 3^n - factorial experiment, we are to follow the following steps.

Step 1.

Find the independent key blocks for the factorial effects to be confounded simultaneously using Equation (1). Let these key blocks be denoted by, M_0, represents the level combinations of key block for the first confounded factorial effect and M'_0 represents the level combinations of key block for the second confounded factorial effect.

Step 2.

Find the common elements of level combinations (row vectors) of these two key blocks and form a matrix, can be denoted by B_1, called the key intrablock subgroups of level combinations of a two factorial effects confounded simultaneously of a 3^n factorial experiment. It can be seen that the key intrablock subgroups contains the lowest level combination for all the factors.

Step 3.

Proceeding column wise we will get the second intrablock subgroup by adding the level combination $(\ldots 0\ 1\ 0)$ with each of the elements of key intrablock subgroup. Similarly, by adding the level combination $(\ldots 0\ 2\ 0)$ (if it is absent in the second intrablock subgroup), with each of the elements of key intrablock, we get the third intrablock subgroup.

Step 4.

Proceeding row wise we will get the fourth, fifth and sixth intrablock subgroups by adding the vector $(\ldots 0\ 0\ 1)$ with each of the vector elements of key, second and third intrablock subgroups. In the same manner, we will get the seventh, eighth and ninth intrablock subgroups by adding the vector $(\ldots 0\ 0\ 2)$ with each of the vector elements in the key, second and third intrablock subgroups. Finally, we get all nine intrablock subgroups, each of the intrablock subgroups is an incomplete block of a 3^n - factorial experiment confounded simultaneously with two factorial effects.

The method is illustrated with an example described in the section below.

Example. Suppose we are to construct the layout of 3^4 - factorial experiment where the factorial effects $ABCD$ and $ABCD^2$ are confounded simultaneously.

The plan is given by matrix, $M = [M_0 \quad M_1 \quad M_2]_{27\times 12}$;

$M_u = [V_1\{3^0\}\ \ V_2\{3^1\}\ \ V_3\{3^2\}\ a_u]$; $u = 0, 1, 2$; with

$V_1\{1\} = 1[0I_9\ \ 1I_9\ \ 2I_9]_{27\times 1}$;
$V_2\{3\} = 3[0I_3\ \ 1I_3\ \ 2I_3]_{27\times 1}$ and
$V_3\{9\} = 9[0I_1\ \ 1I_1\ \ 2I_1]_{27\times 1}$.

The adjustment vector, $a_u = [a_{u1}\ a_{u2}\ \cdots\ a_{u27}]'$ could be obtained by solving the symbolic equation correspond to the factorial effect to be confounded. Using the above method (Jalil, M. A. et al), we can find the key block M_0, where the factorial effect $ABCD$ is to be confounded as follows.

Step 1.

The matrices M_0 and M'_0 are the key blocks confounded with $ABCD$ and $ABCD^2$ in 3^4 - factorial experiments.

$$M_0 = \begin{bmatrix} 0&0&0&0\\0&0&1&2\\0&0&2&1\\0&1&0&2\\0&1&1&1\\0&1&2&0\\0&2&0&1\\0&2&1&0\\0&2&2&2\\ \\1&0&0&2\\1&0&1&1\\1&0&2&0\\1&1&0&1\\1&1&1&0\\1&1&2&2\\1&2&0&0\\1&2&1&2\\1&2&2&1\\ \\2&0&0&1\\2&0&1&0\\2&0&2&2\\2&1&0&0\\2&1&1&2\\2&1&2&1\\2&2&0&2\\2&2&1&1\\2&2&2&0 \end{bmatrix} \qquad M'_0 = \begin{bmatrix} 0&0&0&0\\0&0&1&1\\0&0&2&2\\0&1&0&1\\0&1&1&2\\0&1&2&0\\0&2&0&2\\0&2&1&0\\0&2&2&1\\ \\1&0&0&1\\1&0&1&2\\1&0&2&0\\1&1&0&2\\1&1&1&0\\1&1&2&1\\1&2&0&0\\1&2&1&1\\1&2&2&2\\ \\2&0&0&2\\2&0&1&0\\2&0&2&1\\2&1&0&0\\2&1&1&1\\2&1&2&2\\2&2&0&1\\2&2&1&2\\2&2&2&0 \end{bmatrix}$$

Step 2. Select the common vector elements from M_0 and M'_0, we have,

$$B_1=\begin{bmatrix}0&0&0&0\\0&1&2&0\\0&2&1&0\\1&0&2&0\\1&1&1&0\\1&2&0&0\\2&0&1&0\\2&1&0&0\\2&2&2&0\end{bmatrix}$$

Step 3. Add $(0\ 0\ 1\ 0)$ to each of the vector elements of the key intrablock subgroup, we get,

$$B_2=\begin{bmatrix}0&0&1&0\\0&1&0&0\\0&2&2&0\\1&0&0&0\\1&1&2&0\\1&2&1&0\\2&0&2&0\\2&1&1&0\\2&2&0&0\end{bmatrix}$$

To obtain the third intrablock B_3, add the vector $(0\ 0\ 2\ 0)$ to each of the vector elements of the key intrablock subgroup. Thus,

$$B_3=\begin{bmatrix}0&0&2&0\\0&1&1&0\\0&2&0&0\\1&0&1&0\\1&1&0&0\\1&2&2&0\\2&0&0&0\\2&1&2&0\\2&2&1&0\end{bmatrix}$$

Step 4. We can find the fourth block B_4 by adding the vector element $(0\ 0\ 0\ 1)$ with each of the vector elements of the key intrablock B_1; the intrablock B_5 can be found by adding the vector element $(0\ 0\ 0\ 1)$ with each of the vector elements of the second block B_2. Similarly, we can find the sixth block by adding the vector element $(0\ 0\ 0\ 1)$ with each of the vector elements of the third block B_3. Thus,

B_4 is obtained by adding the vector $(0\ 0\ 0\ 1)$ to each of the vector elements of B_1;

B_5 is obtained by adding the vector $(0\ 0\ 0\ 1)$ to each of the vector elements of B_2;

B_6 is obtained by adding the vector $(0\ 0\ 0\ 1)$ to each of the vector elements of B_3; and

B_7 is obtained by adding the vector $(0\ 0\ 0\ 2)$ to each of the vector elements of B_1;

B_8 is obtained by adding the vector $(0\ 0\ 0\ 2)$ to each of the vector elements of B_2;

B_9 is obtained by adding the vector $(0\ 0\ 0\ 2)$ to each of the vector elements of B_3;

Thus, we have the complete layout of nine intrablock subgroups as shown by,

$$\begin{bmatrix}
 & 0\ 0\ 0\ 0 & & 0\ 0\ 0\ 1 & & 0\ 0\ 0\ 2\\
 & 0\ 1\ 2\ 0 & & 0\ 1\ 2\ 1 & & 0\ 1\ 2\ 2\\
 & 0\ 2\ 1\ 0 & & 0\ 2\ 1\ 1 & & 0\ 2\ 1\ 2\\
 & 1\ 0\ 2\ 0 & & 1\ 0\ 2\ 1 & & 0\ 0\ 2\ 2\\
[B_1] & 1\ 1\ 1\ 0 & [B_4] & 1\ 1\ 1\ 1 & [B_7] & 0\ 1\ 1\ 2\\
 & 1\ 2\ 0\ 0 & & 1\ 2\ 0\ 1 & & 0\ 2\ 0\ 2\\
 & 2\ 0\ 1\ 0 & & 2\ 0\ 1\ 1 & & 0\ 0\ 1\ 2\\
 & 2\ 1\ 0\ 0 & & 2\ 1\ 0\ 1 & & 0\ 1\ 0\ 2\\
 & 2\ 2\ 2\ 0 & & 2\ 2\ 2\ 1 & & 0\ 2\ 2\ 2\\
 & & & & & \\
 & 0\ 0\ 1\ 0 & & 0\ 0\ 1\ 1 & & 0\ 0\ 1\ 2\\
 & 0\ 1\ 0\ 0 & & 0\ 1\ 0\ 1 & & 0\ 1\ 0\ 2\\
 & 0\ 2\ 2\ 0 & & 0\ 2\ 2\ 1 & & 0\ 2\ 2\ 2\\
 & 1\ 0\ 0\ 0 & & 1\ 0\ 0\ 1 & & 1\ 0\ 0\ 2\\
[B_2] & 1\ 1\ 2\ 0 & [B_5] & 1\ 1\ 2\ 1 & [B_8] & 1\ 1\ 2\ 2\\
 & 1\ 2\ 1\ 0 & & 1\ 2\ 1\ 1 & & 1\ 2\ 1\ 2\\
 & 2\ 0\ 2\ 0 & & 2\ 0\ 2\ 1 & & 2\ 0\ 2\ 2\\
 & 2\ 1\ 1\ 0 & & 2\ 1\ 1\ 1 & & 2\ 1\ 1\ 2\\
 & 2\ 2\ 0\ 0 & & 2\ 2\ 0\ 1 & & 2\ 2\ 0\ 2\\
 & & & & & \\
 & 0\ 0\ 2\ 0 & & 0\ 0\ 2\ 1 & & 0\ 0\ 2\ 2\\
 & 0\ 1\ 1\ 0 & & 0\ 1\ 1\ 1 & & 0\ 1\ 1\ 2\\
 & 0\ 2\ 0\ 0 & & 0\ 2\ 0\ 1 & & 0\ 2\ 0\ 2\\
 & 1\ 0\ 1\ 0 & & 1\ 0\ 1\ 1 & & 1\ 0\ 1\ 2\\
[B_3] & 1\ 1\ 0\ 0 & [B_6] & 1\ 1\ 0\ 1 & [B_9] & 1\ 1\ 0\ 2\\
 & 1\ 2\ 2\ 0 & & 1\ 2\ 2\ 1 & & 1\ 2\ 2\ 2\\
 & 2\ 0\ 0\ 0 & & 2\ 0\ 0\ 1 & & 2\ 0\ 0\ 2\\
 & 2\ 1\ 2\ 0 & & 2\ 1\ 2\ 1 & & 2\ 1\ 2\ 2\\
 & 2\ 2\ 1\ 0 & & 2\ 2\ 1\ 1 & & 2\ 2\ 1\ 2
\end{bmatrix}$$

It is easy to verify that,

$(B_1 + B_2 + B_3)$ Vs. $(B_4 + B_5 + B_6)$ Vs. $(B_7 + B_8 + B_9)$; confounds the 1st effect, $ABCD$;

$(B_1 + B_4 + B_7)$ Vs. $(B_2 + B_5 + B_8)$ Vs. $(B_3 + B_6 + B_9)$; confounds the 2nd effect, $ABCD^2$;

$(B_1 + B_5 + B_9)$ Vs. $(B_2 + B_6 + B_7)$ Vs. $(B_3 + B_4 + B_8)$; confounds the 1st generalized effect, $ABCD \times ABCD^2 = ABC$; and

$(B_1 + B_6 + B_8)$ Vs. $(B_2 + B_4 + B_9)$ Vs. $(B_3 + B_5 + B_7)$; confounds the 2nd generalized effect, $ABCD \times (ABCD^2)^2 = D$.

Conclusion

In this article, we have introduced a method of simultaneous confounding in a 3^n factorial experiment using matrix algebra. It becomes easier and rewarding than any methods available in the construction of simultaneous confounding of 3^n factorial experiments. The method is restricted to p^n factorial experiment when $p = 3$.

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