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Coupling of Conduction with Laminar Natural Convection Flow from a Vertical Flat Plate

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Abstract

In this paper the thermo-fluid-dynamic field resulting from the coupling of natural convection along and conduction inside a heated flat plate is studied by means of implicit finite difference method in the entire region starting from the lower part of the plate to down stream. The results in terms of shear stress coefficient and surface temperature coefficient for different Prandtl number are compared with the perturbation solution near the lower part of the plate and asymptotic solution in the down stream region and founded in excellent agreement.

Keywords: Natural convection flow, vertical flat plate and coupling conduction

Nomenclature

x	stream wise coordinate	ψ	stream function
y	transverse coordinate	ν	kinematic viscosity
u	velocity component in the x-direction	ρ	fluid density
v	velocity component in the y-direction	μ	viscosity of the fluid
f	dimensionless stream function	b	plate thickness
g	acceleration due to gravity	d	$\beta(T_{b-} T_\infty)$
T	temperature of the fluid	l	length of the plate
T_b	temperature at outside of the plate	κ_s	thermal conductivity of the ambient solid
T_∞	temperature of the ambient fluid	κ_f	thermal conductivity of the ambient fluid
T_{so}	solid temperature	θ	dimensionless temperature
C_p	specific heat	L	reference length , $\nu^{2/3}/g^{1/3}$
Pr	Prandtl number	p	coupling parameter

I. Introduction

It is well established that when convective heat transfer results are strongly dependent on thermal boundary conditions, consideration of convective heat transfer problems as conjugated problems is necessary to obtain physically more strict results. Many research efforts have been given to the conjugate problem of forced convection heat transfer both experimental and theoretical but few work have been devoted to the conjugate problem of free convection. Gdalevich and Fertman [1] stated conclusively that the use of numerical method for solving the initial system of governing partial differential equation such as finite difference methods is evidently the most promising in studies of conjugate free convection. According to Kelleher and Yang [2] the analytical treatments used extensively in conjugate forced convection problems are difficult due to matching a non-linear solution of free convection in a fluid with linear conduction solution in a solid body at the solid fluid interface. Successful analytical solution the problem of conjugate free convection about tapered, downward projection fin of a simple power law form to be obtained by Lock and Gunn [3]. Analytical solutions of conjugate free convection, if it can be obtained may be useful to seize the main future of the conjugate problem and to find the dimensionless parameter which controls the characteristics of the conjugate free convection. Chida and Katto [4] perform studies the conjugate problems in this direction by the use of the vertical dimensional analysis. They applied their method to the interpretation of previously studied conjugate heat transfer problems. When convective heat transfer depends strongly on the thermal boundary conditions natural convection must be studied as a mixed problem if one needs accurate analysis of the thermo-fluid-dynamic field that pointed one Miyamoto et al. [5]. By this authors and analysis has been given showing the relative importance of the parameters of the problem with reference with axial heat conduction. Timma and Padet [6] by extending the analysis of Gosse [7] have developed a technique which improves the results given by the first term of asymptotic solution of the problem. In the same work a new correlation for the evaluation of heat transfer coefficient had also been presented. This analysis holds for high value of the abscissa of x; the value of the point x_0 from which the expansion is valid depends on the parameters that govern the problem. Further contributions to the study of coupled natural convection by evaluating by the region which the point x_0 falls in, has been made by Pozzi and Luppo [8], improving the results concerning the asymptotic expansion by adding terms of higher order with respect to the first one discussed the general form of the asymptotic expansions which is singular for the presence of eigen solutions and determined the expansion holding for small values of x in an accurate way, by taking into account many terms of the series by means of paddy approximant techniques.

In the present paper we have used to give further contribution to the study of coupled natural convection introducing new transformation to the problem posed by Pozzi et al. [8] that leads the solution for x raise between 0 to ∞ that is solution near the leading edge to down stream region at the vertical surface.

II. Equations Governing the Flow

In order to describe the steady two-dimensional flow due to free convection flow along a side of a vertical flat plate of thickness b insulated on the edges and with a temperature T_b maintained on the other side (Fig. 1) one must solve the coupled thermal fields in the solid and in the fluid. The coupling conditions require that the temperature and heat flux be continuous at the interface.

The thermo-fluid-dynamic field in the fluid is governed by the boundary layer equations which are given as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu_f\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty) \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\kappa_f}{\rho c_p}\frac{\partial^2 T}{\partial y^2} \tag{3}$$

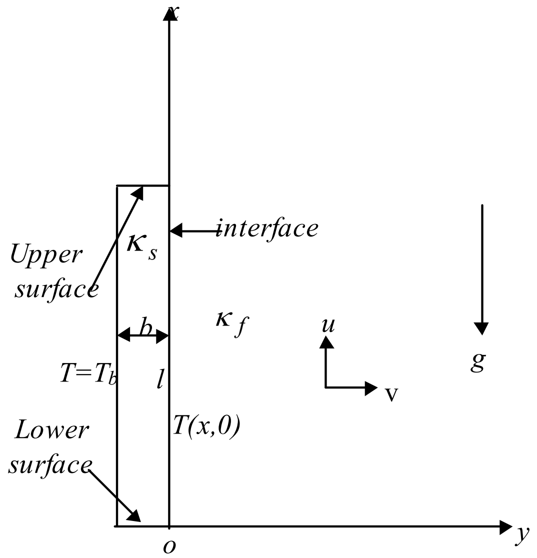

Fig. 1. A vertical flat plate and the coordinate system

where u and v are the velocity components along the x and y directions respectively, T is the temperature of the fluid, g is the gravitational acceleration, β is the volumetric coefficient of thermal expansion, ν is the kinematic viscosity, ρ is the density , k_f is the thermal conductivity of the ambient fluid, C_p is the specific heat with constant pressure.

The boundary conditions to be satisfied by the above equations

$$\begin{aligned} &u=0,\ v=0 \quad \text{at} \quad y=0\\ &u=0,\ T=T_\infty \ \text{at} \quad y=\infty \end{aligned} \tag{4}$$

At the interface

$$\kappa_s\frac{\partial T_{so}}{\partial y}=\kappa_f\left(\frac{\partial T}{\partial y}\right)_{y=0} \tag{5}$$

when k_s is the thermal conductivity of the solid and the temperature T_{so} in the solid as given by Miamoto et al. [5] is

$$T_{so}=T(x,0)-[T_b-T(x,0)]\frac{y}{b} \tag{6}$$

where $T(x,0)$ is the unknown temperature at the interface to be determined from the solutions of Equations (1)-(3).

Now we introduce the following dimensionless dependent and independent variables:

$$\begin{aligned} &\bar{x}=\frac{x}{L},\bar{y}=\frac{y}{L}d^{1/4},u=\frac{\nu}{L}d^{1/2}\bar{u},\\ &v=\frac{\nu}{L}d^{1/4}\bar{v},\ \frac{T-T_\infty}{T_b-T_\infty}=\theta,\\ &L=\frac{\nu^{2/3}}{g^{1/4}},\quad d=\beta(T_b-T_\infty) \end{aligned} \tag{7}$$

into the Equations (1)-(3) get the following dimensionless equations:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{8}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\partial^2 u}{\partial y^2}+\theta \tag{9}$$

$$u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}=\frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2} \tag{10}$$

The corresponding boundary conditions (4) and (5) take the form

$$\begin{aligned} &u=v=0,\ \theta-1=p\frac{\partial\theta}{\partial y}\ \text{at}\ y=0\\ &u=\theta=0 \quad \text{at}\ y=\infty \end{aligned} \tag{11}$$

where $\Pr=\mu c_p/\kappa$ and $p=(\kappa_f/\kappa_s)(b/L)d^{1/4}$ are the Prandtl number and the coupling parameter respectively. The problem described here governed by the coupling parameter p, the order of magnitude of which depends essentially on b/L and κ_f/κ_s, $d^{1/4}=O(1)$. As L is small, b/L is very large on. When the fluid is air $\kappa_f/\kappa_s<<1$ if this plate is highly conductive i.e. $\kappa_s>>1$ and reach the order of 0.1 for materials such as glass. Therefore p is in many cases, but not always, a small number. Briefly in the present investigation we have considered as $p=1$ which is accepted for b/L of $O(\kappa_f/\kappa_s)$.

Accordingly Pozzi and Lupo [8], for the downstream and upstream regimes introduced the following transformations respectively:

$$\psi=x^{3/4}f(x,\eta),\eta=yx^{-1/4},\theta=g \tag{12}$$

and

$$\psi=x^{4/5}F(x,\eta),\eta=yx^{-1/5},\theta=x^{1/5}G \tag{13}$$

Here ψ is the stream function that satisfies the Equation of continuity and η is pseudo-similarity variables.

Combining the transformations given by (12) and (13) following generalized transformation for the flow region starting from upstream to downstream can be determined. The new transformations are as follows:

$$\psi = x^{4/5}(1+x)^{-1/20} f(\eta, x),$$
$$\eta = yx^{-1/5}(1+x)^{-1/20}, \qquad (14)$$
$$\theta = x^{1/5}(1+x)^{-1/5} h$$

Now using transformations given into (14) in (8)-(10) we get

$$f''' + \frac{16+15x}{20(1+x)} ff'' - \frac{6+5x}{10(1+x)} f'^2 + h = x\left(f' \frac{\partial f'}{\partial x} - f'' \frac{\partial f}{\partial x}\right) \qquad (15)$$

$$\frac{1}{P_r} h'' + \frac{16+15x}{20(1+x)} fh' - \frac{1}{5(1+x)} f'h = x\left(f' \frac{\partial h}{\partial x} - h' \frac{\partial f}{\partial x}\right) \qquad (16)$$

In the above equations the primes denote differentiation with respect to the η

Boundary conditions (11) then take the following form

$$f(x,0) = f'(x,0) = 0,$$
$$h'(x,0) = -(1+x)^{1/4} + x^{1/5}(1+x)^{1/20} h(x,0) \qquad (17)$$
$$f'(x,\infty) = 0, \quad h(x,\infty) = 0$$

In this paper we find the solutions of the Equations (15)-(17) by employing the implicit finite difference method together with the Keller-box scheme for different values of the pertinent parameter the Prandtl number. Here we avoid repetition of the detailed discussion of this method since this had already been discussed elaborately by Cebeci and Bradshow [13] and later by Hossain et al. [9]

III. Results and Discussion

If we know the values of the functions $f(\eta, x)$, $h(\eta, x)$ and their derivatives for different values of the Prandtl number Pr, we may calculate the numerical values of the surface temperature $\theta(0, x)$ and the velocity gradient $f''(0, x)$ at the surface, which are important from the physical point of view. Numerical values of $\theta(0, x)$ are obtained from the following relations:

$$\theta(0, x) = x^{1/5} (1+x)^{-1/5} h(x, 0)$$

Numerical value of the velocity gradient $f''(0, x)$, and the surface temperature $\theta(0, x)$, are depicted graphically in Figs. 2 and 3 against the axial distance x in the interval [0,10] for the values the Prandtl number Pr = 0.73, 1.97 and 2.97 that had been taken into account by Pozzi and Lupo [8] in their analyses. In Fig. 2 one can seen that an increase of Prandtl number leads to decrease of the value of shear stress coefficient $f''(0, x)$ as well the surface temperature $\theta(0, x)$.

In Figs. 4 and 5, numerical value of velocity profile $f'(\eta, x)$ and temperature profile $\theta(\eta, x)$, are depicted against the similarity variable η in [0, 8] are shown graphically for values of the Prandtl number Pr = 0.73, 1.97 and 2.97. In the above Figures effect of the axial distance are also shown. To show its effect on the velocity and the temperature profiles in the boundary layer regimes value of x are chosen to be 1.05 and 3.33 which are represented by the solid and broken curves. From Fig. 4 one may observe that both the velocity and the temperature profiles decrease owing to increase in the value of the Prandtl number. We may also observe that an increase in the value of the axial distance x leads to decrease in both the velocity and the temperature profiles. In order to compare the present results with those of Ref. [8] for small and large values of x we discuss the method of solutions in details:

Solution for small x

For small x Equations (15)-(17) can be written to the following form

$$f''' + \frac{4}{5} ff'' - \frac{3}{5} f'^2 + h = x\left(f' \frac{\partial f'}{\partial x} - f'' \frac{\partial f}{\partial x}\right) \qquad (18)$$

$$\frac{1}{\mathrm{Pr}} h'' + \frac{4}{5} fh' - \frac{1}{5} f'h = x\left(f' \frac{\partial h}{\partial x} - h' \frac{\partial f}{\partial x}\right) \qquad (19)$$

with conditions

$$f(x,0) = f'(x,0) = 0,$$
$$h'(x,0) = -1 + x^{1/5} h \qquad (20)$$
$$f'(x,\infty) = h(x,\infty) = 0$$

Since x is small, we expand the functions $f(x, \eta)$ and $h(x, \eta)$ in powers of $x^{1/5}$, as given below:

$$f(x,\eta) = \sum_{i=0}^{\infty} x^{i/5} f_i(\eta)$$
$$\text{and } h(x,\eta) = \sum_{i=0}^{\infty} x^{i/5} h_i(\eta) \qquad (21)$$

Now, substituting the expansions (21) into Equations (18)-(20) and taking the terms $O(x^0)$ and $O(x^{n/5})$ respectively we get

$$f_0''' + \frac{4}{5} f_0 f_0'' - \frac{3}{5} f_0'^2 + h_0 = 0 \qquad (22)$$

$$\frac{1}{\mathrm{Pr}} h_0'' + \frac{4}{5} f_0 h_0' - \frac{1}{5} f_0' h_0 = 0 \qquad (23)$$

$$f_0(0) = f_0'(0) = 0, \; h_0'(0) = -1$$
$$f_0'(\infty) = 0, \; h_0(\infty) = 0 \qquad (24)$$

$$f_n''' + \frac{4}{5} \sum_{k=0}^{n} f_k f_{n-k}'' - \frac{3}{5} \sum_{k=0}^{n} f_k' f_{n-k}' + h_k = \sum_{k=1}^{n} \frac{k}{5} \left(f_{n-k}' f_k' - f_{n-k}'' f_k \right) \qquad (25)$$

$$\frac{1}{P_r} h_n'' + \frac{4}{5} \sum_{k=0}^{n} f_k h_{n-k}' - \frac{1}{5} \sum_{k=0}^{n} f_{n-k}' h_k = \sum_{k=1}^{n} \frac{k}{5} \left(f_{n-k}' h_k - h_{n-k}' f_k \right) \qquad (26)$$

$$f_n(0) = f_n'(0) = 0, \; h_n' = h_{n-1} (n > 0) \qquad (27)$$
$$f_n'(\infty) = h_n(\infty) = 0$$

Equations (22)-(24) are coupled and nonlinear that had already been integrated by Sparrow and Gregg [10] for the natural convection flow from vertical flat plate with uniform surface heat flux with little difference in the coefficients. Hence here we again obtain the solutions using the method of iteration developed by Nachtsheim and Swigert [14] for different values of the Prandtl number considered before. Subsequent equations for $n = 1,2,3,\cdots\cdots$ are coupled and with non-homogeneous boundary conditions nontrivial solutions of which can be obtained easily using the aforementioned method. Here we have obtained the solutions $n = 10$. Numerical values of $f_i''(0)$ and $\theta_i(0)$ thus obtained are entered in Table 1 and Table 2 respectively and are compared with those obtained by Pozzi and Lupo [8]. The comparison shows excellent agreement between these two results.

After knowing the values of the functions $f(\eta,x)$, $h(\eta,x)$ and their derivatives we can calculate the values of the local skin friction coefficient $f''(0,x)$ and the surface temperature coefficient $\theta(0,x)$ in the region near the point leading edge from the following relations:

$$f''(0,x) = x^{2/5}(f_0''(0) + x^{1/5}f_1''(0) + x^{2/5}f_2''(0) + x^{3/5}f_3''(0) + ...) \tag{28a}$$

$$\theta(0,x) = x^{1/5}(h_0(0) + x^{1/5}h_1(0) + x^{2/5}h_2(0) + x^{3/5}h_3(0) + ...) \tag{28b}$$

Numerical values obtained from the above expression for $f''(0,x)$ and $\theta(0,x)$ are shown graphically in Fig. 2 and Fig.3 respectively for different values of x as well as Pr. The broken curves for smaller values of x are the representations of these solutions. The comparisons of these curves with the solids that are obtained by the finite difference are found in excellent agreement.

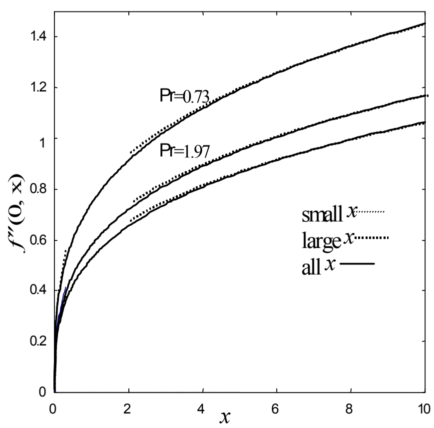

Fig. 2. Skin friction against axial distance x for different Pr.

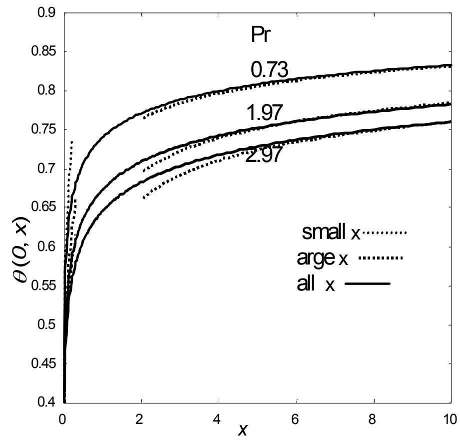

Fig. 3. Non dimensional temperature against axial distance x for different Pr.

Solutions for large x

For large x, taking the limits of the coefficients of Equations (15)-(17) as $x \to \infty$, the equations reduce to

$$f''' + \frac{3}{4}ff'' - \frac{1}{2}f'^2 + h = x\left(f'\frac{\partial f'}{\partial x} - f''\frac{\partial f}{\partial x}\right) \tag{29}$$

$$\frac{1}{Pr}h'' + \frac{3}{4}fh' = x\left(f'\frac{\partial h}{\partial x} - h'\frac{\partial f}{\partial x}\right) \tag{30}$$

Corresponding boundary conditions then take the form

$$f(x,0) = f'(x,0) = 0, \quad h(x,0) = 1 + x^{-1/4}h' \\ f'(x,\infty) = h(x,\infty) = 0 \tag{31}$$

Since x is large, solutions of the Equations (29)-(31) may be obtained by using the perturbation method. We expand the functions $f(x,\eta)$ and $h(x,\eta)$ in powers of $x^{-1/4}$ as given below:

$$f(x,\eta) = \sum_{i=0}^{\infty} x^{-i/4} f_i(\eta) \\ \text{and } h(x,\eta) = \sum_{i=0}^{\infty} x^{-i/4} h_i(\eta) \tag{32}$$

Now substituting the expansions (32) in Equations (29)-(31) and taking the terms $O(x^0)$ and $O(x^{-n/4})$ we get

$$f_0''' + \frac{3}{4}f_0f_0'' - \frac{1}{2}f_0'^2 + h_0 = 0 \tag{33}$$

$$\frac{1}{Pr}h_0'' + \frac{3}{4}f_0h_0' = 0 \tag{34}$$

$$f_0(0) = f_0'(0) = 0, \quad h_0(0) = 1 \\ f_0'(\infty) = 0, \quad h_0(\infty) = 0 \tag{35}$$

$$f_n''' + \frac{3}{4}\sum_{k=0}^{n} f_k f_{n-k}'' - \frac{1}{2}\sum_{k=0}^{n} f_k' f_{n-k}' + h_n = \sum_{k=1}^{n} \frac{k}{4}\left(f_{n-k}'' f_k - f_{n-k}' f_k'\right) \tag{36}$$

$$\frac{1}{\Pr}h_n'' + \frac{3}{4}\sum_{k=0}^{n} f_k h_{n-k}' = \sum_{k=1}^{n}\frac{k}{4}\left(h_{n-k}' f_k - f_{n-k}' h_k\right) \tag{37}$$

$$f_n(0) = f_n'(0) = 0, \quad h_n = h_{n-1}'(n > 0) \tag{38}$$
$$f_n'(\infty) = h_n(\infty) = 0$$

Equations (33)-(34) along with the boundary conditions (35) represent the similarity equations governing the natural convection flow from a vertical heated surface maintained at uniform temperature that had first been investigated by Pohlhausen and Schmidt [11,12]. As before, these equations also differ only with the coefficients with those deduced by the aforementioned authors. However for comparison purpose here also we implement the Nachtsheim and Swigert [14] iteration technique in finding the solutions for different values of the Prandtl number. As before here also we apply the same method, unlike the Eigen value solution obtained by Pozzi and Lupo [8], in finding the solutions of the subsequent sets of equations for $n = 1,2,3,\cdots\cdots$; since the boundary condition of the above equations are non-homogeneous and we obtained non-trivial solutions easily. In Table 3 numerical values of the functions $f_i''(0)$ and $\theta_i(0)$ (for $i = 1, 2$ and 3) are entered the for different values of the Prandtl number and compared with the corresponding numerical values obtained by Pozzi and and lupo [8]. Here also the comparison between these two results is found in excellent agreement.

Table. 1. Initial expansion: values of $f_n''(0)$ for comparison

n	Pr = 0.733		Pr = 2.97	
	Present	Pozzi et al	Present	Pozzi et al
0	1.5366	1.540	9.1705×10^{-1}	9.197×10^{-1}
1	-1.64625	-1.641	-6.8224×10^{-1}	-6.799×10^{-1}
2	1.62421	1.624	4.7042×10^{-1}	4.698×10^{-1}
3	-1.37008	-1.371	-2.7872×10^{-1}	-2.787×10^{-1}
4	9.4450×10^{-1}	9.453×10^{-1}	1.3590×10^{-1}	1.360×10^{-1}
5	-4.834×10^{-1}	-4.840×10^{-1}	-4.981×10^{-2}	-4.992×10^{-2}
6	1.2086×10^{-1}	1.210×10^{-1}	9.43×10^{-3}	9.470×10^{-3}
7	7.296×10^{-2}	7.296×10^{-2}	3.30×10^{-3}	3.295×10^{-3}
8	-1.091×10^{-1}	-1.095×10^{-1}	-3.88×10^{-3}	-3.895×10^{-3}
9	5.675×10^{-2}	5.699×10^{-2}	1.56×10^{-3}	1.570×10^{-3}
10	7.51×10^{-3}	7.548×10^{-3}	3.0×10^{-5}	3.10×10^{-5}

Table. 2. Initial expansion: values of $\theta_n(0)$ for comparison

n	Pr = 0.733		Pr = 2.97	
	Present	Pozzi et al	Present	Pozzi et al
0	2.04182	2.042	1.41297	1.411
1	-3.08578	-3.083	-1.48339	-1.481
2	3.79145	3.789	1.271	1.271
3	-3.88758	-3.886	-9.1538×10^{-1}	-9.147×10^{-1}
4	3.32163	3.322	5.5127×10^{-1}	5.512×10^{-1}
5	-2.29624	-2.298	-2.7025×10^{-1}	-2.704×10^{-1}
6	1.17032	1.172	9.876×10^{-2}	9.896×10^{-2}
7	-2.848×10^{-1}	-2.853×10^{-1}	-1.817×10^{-2}	-1.827×10^{-2}
8	-1.838×10^{-1}	-1.844×10^{-1}	-6.96×10^{-3}	-6.959×10^{-3}
9	2.6714×10^{-1}	2.681×10^{-1}	7.84×10^{-3}	7.875×10^{-3}
10	-1.3567×10^{-1}	-1.362×10^{-1}	-3.07×10^{-3}	-3.093×10^{-3}

Table. 3. Asymptotic expansion: values of $f_i''(0)$ for comparison

n	Pr = 0.733		Pr = 2.97	
	Present	Pozzi et al	Present	Pozzi et al
0	9.5320×10^{-1}	9.532×10^{-1}	7.5282×10^{-1}	7.522×10^{-1}
1	-2.949×10^{-1}	-2.908×10^{-1}	-3.7057×10^{-1}	-3.693×10^{-1}
2	1.1481×10^{-1}	1.143×10^{-1}	2.3914×10^{-1}	2.392×10^{-1}
3	-4.121×10^{-1}	-4.128×10^{-1}	-1.5042×10^{-1}	-1.515×10^{-1}

Table. 4. Asymptotic expansion: values of $\theta_n'(0)$ for comparison

n	Pr = 0.733		Pr = 2.97	
	Present	Pozzi et al	Present	Pozzi et al
0	-3.610×10^{-1}	-3.591×10^{-1}	-5.7446×10^{-1}	-5.749×10^{-1}
1	1.3193×10^{-1}	1.315×10^{-1}	3.4114×10^{-1}	3.414×10^{-1}
2	-3.616×10^{-2}	-3.593×10^{-2}	-1.5474×10^{-1}	-1.545×10^{-1}
3	3.847×10^{-8}	3.845×10^{-8}	8.481×10^{-7}	8.482×10^{-7}

Finally, knowing the values of the functions $f(\eta,x)$, $h(\eta,x)$ and their derivatives we can calculate the values of the local friction coefficient and the temperature for large values of x in the region where the flow is dominated only by the buoyancy force:

$$f''(0,x) = f_0''(0) + x^{-1/4} f_1''(0) + x^{-2/4} f_2''(0) + x^{-3/4} f_3''(0) + \ldots \tag{39a}$$

and

$$\theta(0,x) = h_0(0) + x^{-1/4} h_1(0) + x^{-2/4} h_2(0) + x^{3/4} h_3(0) + \ldots \tag{39b}$$

In Fig. 2 and Fig. 3 numerical values of $f''(0,x)$ and $\theta(0,x)$ are depicted and compared with the finite difference solutions. It can be seen, the curves shown from these solutions are over lapping with those of the finite difference at large values of x that implies that both the solutions are in excellent agreement

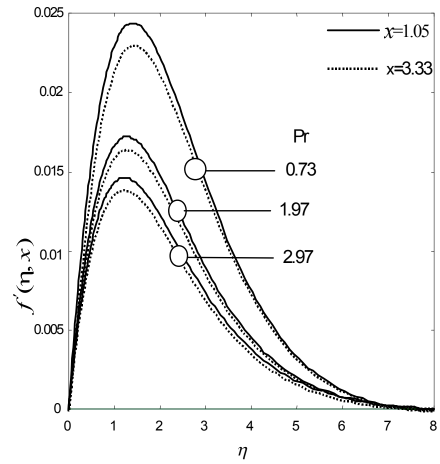

Fig. 4. Velocity profile against η for different Pr.

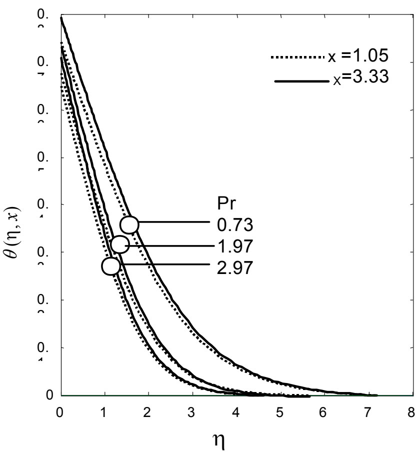

Fig. 5. Temperature profile against η for different Pr.

IV. Conclusions

In the present paper we have contributed to the study of coupled natural convection and conduction in a flat plate introducing a new class of transformation that leads the solution to the regime near the leading edge to down stream regime along the vertical surface. The coupling of conduction required that the temperature and the heat flux be continuous at the interface. The equations are integrated using the finite difference method and the results are found to be in excellent agreement with the corresponding asymptotic solution. We may thus conclude that the present strained coordinate transformations yield appropriate equations results of which more accurate then the perturbation solutions.

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