Dhaka Univ. J. Sci. 58(1): 7-11 2010 (January)

Two Dimensional Continuous Time Markov Chain for Packet Traffic: A Newly Proposed Approach

[1]Md. Imdadul Islam, [1]J. K. Das and [2]M. H. Rahman

[1]*Department of Computer Science and Engineering, Jahangirnagar University, Savar, Dhaka-1342, Bangladesh*

[2]*Department of Applied Physics, Electronics and Communication Engineering, Dhaka University, Dhaka -1000, Bangladesh*

Received on 04. 09. 2007. Accepted for Publication on 27.05.2009

Abstract

Packet traffic is analyzed based on two states continuous time Markovian chain where one state indicates overloaded and the other is under loaded condition of the network. The sojourn time of a state follows exponential distribution. Still no specific Markov chain exists for analysis of two dimensional packet traffic of combined link. In this paper a two dimentinal Markovian chain is proposed for two different types of packet traffic on a common trunk link. Several methods are prevalent to solve such Markovian chain; this paper gives a method based on Laplace Transform to solve the chain. Theoretical explanation of the paper will be helpful for a network planar to allocate channels to support offered traffic of a network.

Key words: Markovian chain, packet traffic, probability transition matrix, coefficient of variation and CAS.

I. Introduction

Traffic offered to a system is called "multidimensional" when it is categorized into different types or classes according to arrival process, service time distribution, bandwidth requirement, quality of service (QoS) requirement, impact of number of users, priority etc. If a communication network is in statistical equilibrium, it can be modeled by a special state transition diagram known as Markovian chain. A finite Markov chain or Markov process describes a closed system in which objects move from one state to another by a pre-determined probability [1]-[4]. Voice traffic of circuit switch network follows Poisson's probability density function (pdf), which can be modeled by finite sate Markovian chain is independent of time. Node or cut equations can be applied to determine different probability sates in normalized form. Multidimensional voice traffic can also be solved using triangular matrix or tabular form shown in [5]. In case of packet traffic (ATM packet, TCP-IP packet, MPEG video packet etc.) inter-arrival time is exponential but service time is deterministic; hence finite state Markovian chain is not applicable for such traffic summarized in [6]. It is always convenient to analyze packet traffic using two state continuous Markovian chain where one is overloaded and the other state is under-loaded state of the network. Any packet arrival in under-loaded sate will be served and that of at over-loaded will be lost. Four traffic parameters i) packet arrival rate at overloaded sate ii) packet arrival rate at under-loaded sate iii) the rate of transition from overloaded to under-loaded sate iv) the rate of transition from under-loaded to overloaded sate, are enough to describe performance of the network [7]-[9]. Above parameters are determined from the traffic histogram of the network after long time observation. Coefficient of variation of interarrival of such traffic is greater than unity reveals some random attitude. To alleviate the situation some intermediate states are incorporated with the underloaded sate of MAP(2) (two sates Markovian arrival process). For proposed two dimensional packet traffic case, same result is found without introducing any additional sate.

Section-2 of the paper gives a mathematical model to solve continuous Markovian chain, section-3 gives the proposed traffic model, section-4 depicts the results of the paper and finally section-5 concludes the entire paper.

II. Mathematical Model

This section depicts a mathematical model to determine time dependent probability transition matrix of a Markovian chain [10]-[14]. In each case, the transition density matrix, A, should have negative diagonal elements and non-negative off-diagonal elements. Each element of A corresponds to transition at a rate from one state to another. Let us consider the simplest case of state transition diagram of two states like fig. 1.

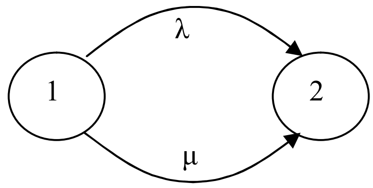

Fig.1. Two sate Markovian chain

The transition density matrix, $A=\begin{bmatrix}-\lambda & \lambda \\ \mu & -\mu\end{bmatrix}$ and the differential equation, **P′ (t)=P(t)A** is used to determine each probability sate. In this method a matrix **(sI-A)**$^{-1}$ is determined, where s is the independent variable of Laplace transform and I is an identity matrix. Finally inverse Laplace transform is taken on each element of the matrix to get the probability states $P_{ij}(t)$.

Now, $$sI-A=\begin{bmatrix}s-\lambda & -\lambda \\ -\mu & s+\mu\end{bmatrix} \quad (1)$$

Characteristics equation,

$$(s+\lambda)(s+\mu)-\lambda\mu=0 \Rightarrow s\{s+(\lambda+\mu)\}=0 \quad (2)$$

$$\therefore (sI-A)^{-1} = \begin{bmatrix} \frac{s+\mu}{s\{s+(\lambda+\mu)\}} & \frac{\lambda}{s\{s+(\lambda+\mu)\}} \\ \frac{\mu}{s\{s+(\lambda+\mu)\}} & \frac{s+\lambda}{s\{s+(\lambda+\mu)\}} \end{bmatrix} \quad (3)$$

$$P_{11}(s) = \frac{s+\mu}{s\{s+(\lambda+\mu)\}} = \left(\frac{\mu}{\lambda+\mu}\right)\frac{1}{s} + \left(\frac{\lambda}{\lambda+\mu}\right)\frac{1}{s+\lambda+\mu} \quad (4)$$

Taking inverse Laplace transform on (4),

$$P_{11}(t) = \frac{\mu}{\lambda+\mu} + \frac{\lambda}{\lambda+\mu} e^{-(\lambda+\mu)t} \quad (5)$$

Similarly, $P_{12}(s) = \frac{\lambda}{s\{s+(\lambda+\mu)\}} = \left(\frac{\lambda}{\lambda+\mu}\right)\left(\frac{1}{s} + \frac{1}{s+\lambda+\mu}\right)$ (6)

Taking inverse Laplace transform on (6),

$$P_{12}(t) = \frac{\lambda}{\lambda+\mu}\left[1 - e^{-(\lambda+\mu)t}\right] \quad (7)$$

III. Proposed Model

In a mixed traffic network call admission scheme (CAS) is incorporated to balance the impact of voice and data traffic on allocated channels. Fig.2 shows the simplest call admission scheme for voice data integrated service where n out of $n+k$ channels are used for combined traffic and the rest k channels are for data traffic based on [15]-[17]. Here A_d and A_v are the offered traffic of data and voice respectively. Any voice call arrival beyond utilization of n channels will be dropped because the rest k channels are exclusively kept for data traffic. The Markovian chain for the CAS of fig.2 is shown in fig.3.

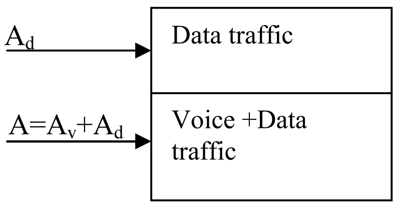

Fig. 2. Call admission scheme of voice data integrated traffic

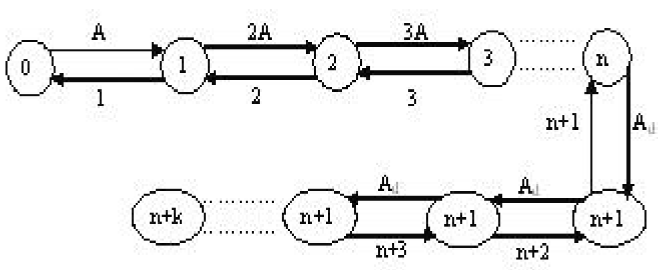

Fig. 3. Markov Chain of fig. 2

All the probability sates $i \le n+k$ are considered as the under-loaded condition of data traffic and any arrival at $n+k$ state it considered as the overloaded sate of the data traffic. Applying cut equations on fig.3 the probability sates are:

$$P_x = \frac{\frac{Ax}{x!}}{\sum_{i=0}^{n}\frac{A^i}{i!} + \frac{A^n}{n!}\sum_{j=1}^{k} A_d^j \prod_{r=1}^{j}\frac{1}{(n+r)}} \;;\; x \le n \quad (8)$$

$$P_x = \frac{\frac{A^n}{n!} A_d^x \prod_{r=1}^{x}\frac{1}{(n+r)}}{\sum_{i=0}^{n}\frac{A^i}{i!} + \frac{A^n}{n!}\sum_{j=1}^{k} A_d^j \prod_{r=1}^{j}\frac{1}{(n+r)}} \;;\; n < x \le n+k \quad (9)$$

All of above sates are considered as underloaded state of the network but any arrival at P_{n+k} will take the network in overloaded state. Such analysis is independent of time and inclusion of time requires time dependent form of Markov chain like previous section. The entire chain can be represented into its equivalent form of two states like fig.4. where sate1 is the overloaded state and any arrival of packet at the rate of λ_1 will be lost [18]-[20]. The rate of transition from overloaded state to under loaded sate is x_1. State-2 is the under loaded sate where any arrival of packet at the rate of λ_2 will be served but has the tendency to back to overloaded state at a rate of x_2. The transition density matrix of fig.4 will be:

$A = \begin{bmatrix} -\lambda_1 - x_1 & x_1 \\ x_2 & -\lambda_2 - x_2 \end{bmatrix}$ and its diagonal matrix, $D = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$.

Kth moment G_k, of interarrival time is expressed like [21],

$$G_k = k!\mathbf{P}(-\mathbf{C})^{-(k+1)}\mathbf{D}\mathbf{e} \quad (10)$$

where, **P** is the initial probability vector and **e** is a unit vector.

Coefficient of variation C_v is: $C_v = \frac{\sqrt{G_2 - G_1^2}}{G_1}$ (11)

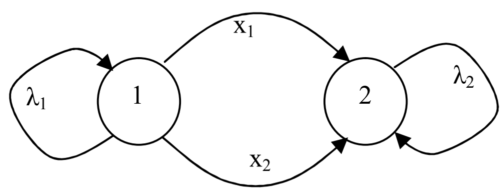

Fig. 4. State transition of packet traffic

Such data traffic model, which is prevalent now-a-days, can be solved quite comfortably. The situation becomes cumbersome for two dimensional data traffic. Here four different probability sates and six pair of transitions are possible taking combinations of states of the traffics. All the four states have self transition depicted by loops of fig.4. For two dimensional packet traffic the state transition chain will take the form of fig.3. Here any probability state

$(i \in \{0, 1\},\ j \in \{0, 1\})$ indicates the loading condition of the traffics like [22]. Flag 0 and 1 indicate under-loaded and over-loaded sate of traffic; i and j are the sates of first and second types of traffic shown in table-1

Table-1

States	Conditions of traffic
(0, 0)	Both traffics are in underloaded state.
(0, 1)	Type-1 in underloaded and type-2 in overloaded state.
(1, 0)	Type-1 in overloaded and type-2 in underloaded state
(1, 1)	Both traffics are in overloaded state

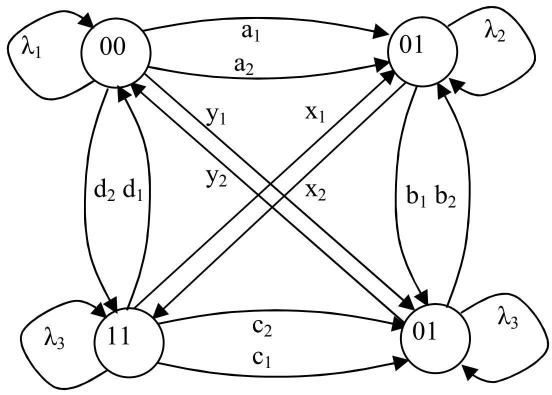

Fig.5. State transition of two dimensional packet traffic

The transition density matrix of fig.5 will be:

$A=$

$$\begin{bmatrix} -\lambda_1-a_1-y_1-d_2 & a_1 & y_1 & d_2 \\ a_2 & -\lambda_2-a_2-b_1-x_2 & b_1 & x_2 \\ y_2 & b_2 & -\lambda_3-b_2-c_1-y_2 & c_1 \\ d_1 & x_1 & c_2 & -\lambda_4-c_2-d_1-x_1 \end{bmatrix} \quad (12)$$

From the relation, **P′(t)=P(t).A**

$$(D+R)P_{11}(t)=a_2P_{12}(t)+y_2P_{13}(t)+d_1P_{14}(t) \quad (13)$$

$$(D+Z)P_{12}(t)=a_1P_{11}(t)+b_2P_{13}(t)+x_1P_{14}(t) \quad (14)$$

$$(D+X)P_{13}(t)=y_1P_{11}(t)+b_2P_{12}(t)+c_1P_{14}(t) \quad (15)$$

$$(D+Y)P_{14}(t)=d_2P_{11}(t)+x_2P_{12}(t)+c_1P_{13}(t) \quad (16)$$

where, $R=\lambda_1+a_1+y_1+d_2$, $Z=\lambda_2+a_2+b_1+x_2$, $X=\lambda_3+b_2+c_1+y_2$ and $Y=\lambda_{2\backslash 4}+c_2+d_1+x_1$

Eliminating $P_{11}(t)$ from (13)-(16),

$$(D^2+aD+b)P_{12}(t)=$$
$$(b_2D+c)P_{13}(t)+(x_1D+d)P_{14}(t) \quad (17)$$

$$(D+e)P_{13}(t)=(y_1/a_1D+f)P_{12}(t)+gP_{14}(t) \quad (18)$$

$$(D+h)P_{14}(t)=(d_2/y_1D+i)P_{13}(t)+jP_{12}(t \quad (19)$$

where, $a=R+Z$, $b=RZ-a_1a_2$, $c=Rb_2+y_2$, $d=x_1R$, $e=X+y_1b_2/a_1$, $f=y_1Z/a_1+b_2$, $g=c_1-y_1x_1/a_1$, $h=Y+d_2c_1/y_1$, $i=d_2X/y_1+c_1$, and $j=x2-d_2b_2/y_1$

Successive elimination of variables will provide fourth order differential equation of $P_{11}(t)$, $P_{12}(t)$, $P_{13}(t)$ and $P_{14}(t)$. Final solution of the differential equations can be obtained with the method of section-2. Now kth moment G_k and co-efficient of variation C_v can be evaluated from equations (10) and (11).

IV. Results

For the Markovian chain of fig.1, the transition matrix $\mathbf{P(t)} = \begin{bmatrix} P_{11}(t) & P_{12}(t) \\ P_{21}(t) & P_{22}(t) \end{bmatrix}$ can be evaluated by any one of six techniques of section-2. The profile of $P_{11}(t)$ and $P_{12}(t)$ of **P(t)** is shown in fig.6 has reverse trajectory. Finally states will be stable with, $P_{11}=\mu/(\lambda+\mu)=3/5$ and $P_{12}=\lambda/(\lambda+\mu)=2/5$ at $t=\infty$ are also visualized from the same figure. Situation is different for self absorbing Markovian chain of fig.4 and fig.5.

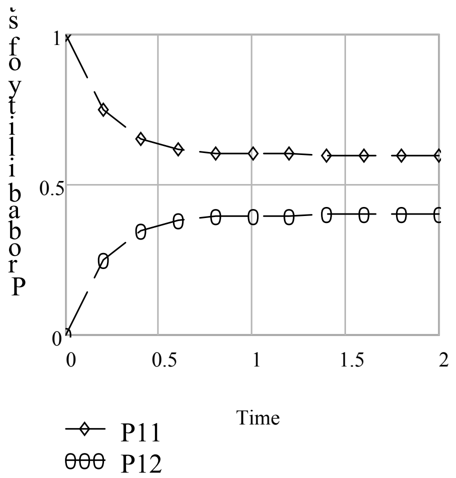

Fig. 6. Probability states of fig.1

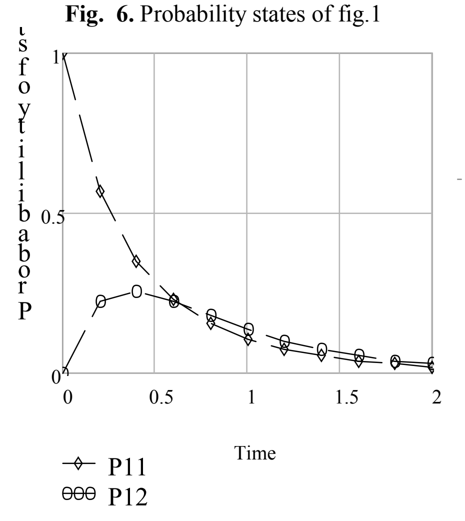

Fig. 7. Probability states of fig.4

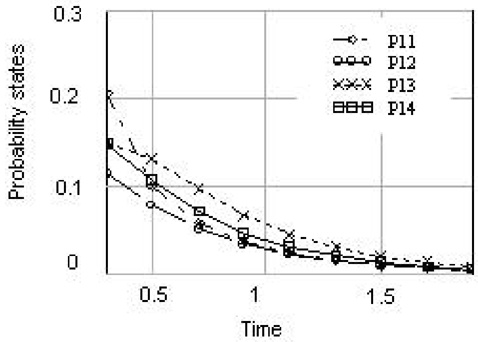

Fig. 8. Probability states of fig.5

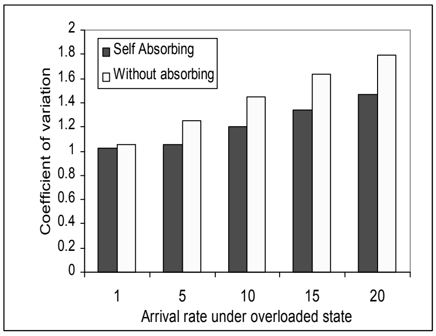

Fig. 9. Coefficient of variation of fig. 2

Trajectory of different probability states of fig.4 ($\lambda_1=1$ packets/unit time, $x_1=2$ packets/unit time, $\lambda_2=2$ packets/unit time, $x_2=1$ packets/unit time) and fig.5 ($\lambda_1=1$, $\lambda_2=2$, $\lambda_3=2$, $\lambda_4=3$, $a_1=2$, $a_2=1$, $x_1=2$, $x_2=4$, $y_1=1.2$, $y_2=1$, $d_1=1$, $d_2=2$, $c_1=1$, $c_2=2$) are shown in fig.7 and fig.8 respectively. All the curves have the tendency of decreasing with time and after some duration they seems to be stable but stability comes after longer time in fig.7 and fig.8 compared to that of fig.6. Incorporation of more states is responsible for such situation.

Fig.9 depicts the comparison of coefficient of variation C_v of self absorbing and non-absorbing cases of two state Markovian chain of fig.4 taking $x_1=2$, $x_2=4$, $\lambda_2=2$. Coefficient of variation is higher for non-absorbing case of course the parameter is higher than unity for both cases. Incorporation of some intermediate states reduces C_v was shown by [21]. The proposed two states Markovian chain deals with four probability states will reduce the wide variation of traffic parameters. The phenomenon is verified in fig.10 where C_v is found less than unity but C_v is more sensitive for the case of non absorbing case like one dimensional Markovian chain.

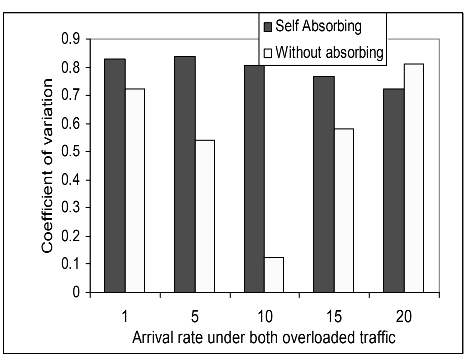

Fig. 10. Coefficient of variation of fig. 3 (λ_2, λ_3, λ_4=0/2)

V. Conclusions

Usually voice traffic of a network is analyzed based on time independent Markovian chain. In case of voice data integrated traffic of recent network above procedure deviates from accuracy. The remedy of the situation is to incorporate M/D/n traffic model but solution of such model becomes troublesome job when n>1 even no solution exits for the case of two dimensional traffic. The proposed traffic model in this paper will be a helping tool for a network planar to select the traffic parameters of the network to ensure desired QoS. Still there is a scope of determination of probability density function (pdf) of the proposed model to compare it with the pdf of other existing traffic models.

----

1. Bijan Jabbari and Woldemar F. Fuhrmann, 1997 "Traffic Modeling and analysis of Flexible Hierarchical Cellular Networks with Speed-Sensitive Handoff Strategy," *IEEE Journal on Selected Areas in Communications,* **15, 8,** 1539-1548.
2. Islam M. I. and S. Hossain, 2003 "An analytical model of traffic performance of mobile cellular network in underlay overlay cell system," *6th International Conference on Computer and Information Technology (ICCIT),* 230-234.
3. Paul Fitzpatrick, Cheng Siong Lee and Bob Warfield, 1997 "Teletraffic Performance of Mobile Radio Networks with Hierarchical Cells and Overflow," *IEEE Journal on selected areas in communications*, **15, 8,** 1549-1557.
4. Kwan L. Yeung and Sanjib Nanda, 1996 "Channel Management in Micro/Macro Cellular Radio Systems," *IEEE Transaction on Vehicular Technology*, **45**, **4**, 14-23.
5. Islam M. I., J. K. Das and S. Hossain, 2007 "Modeling of mixed traffic for mobile cellular network", *Journal of Telecommunications and Information Technology*, National Institute of Telecommunications Szachowa st 104-894 Warsaw, **1/2007,** 83-89.
6. Islam M. I., J. K. Das and S. Hossain , 2006 "A newly designed Markovian chain for packet data traffic", *Jahangirnagar university Journal of Science*, **29**, 63-68.
7. Kamolwun Dankhonsakul and Tapio Erke, 2000 "Resource allocation during handover with dynamic guard bandwidth in wireless ATM," *Proc. of The Third International Symposium on Wireless Personal Multimedia Communications,* **2**, **2**, 579-586.
8. Marichamy P., S. Chakrabarti and S.L. Maskara, 2000 "Rerouting techniques and signaling for handoff in wireless ATM networks", *Proceedings of The Third International Symposium on Wireless Personal Multimedia Communications*, **2**, **2**, 587-592.
9. Chinnasri Suriyadetsakul and Tapio Erke, 2000 "A study of handover performance of variable bit rate sources in a wireless ATM network", *Proceedings of The Third International Symposium on Wireless Personal Multimedia Communications*, **2**, **2**, 593-598.
10. Veerarajan T, Probability, 'Statistics and Random Process' ISBN-0-07-049482-7, chap-9, pp.468-516, Tata McGraw-Hill Publishing Company Limited, 2003
11. Sheldon M. Ross, 2001, 'Introduction to Probability Models,' 7th Edition, ISBN-81-7867-055-0, pp.216-225, Academic Press, A Harcourt Science and Technology Company, San Diego, USA.

12. Mahi Lohi, A. Desgrees du Lou and A.H. Aghvami, 2000 "Service Prioritization Queuing Scheme (SPQS) for Multi-Layer Cellular System*" The third International Symposium on WPMC*, 103-108, Bangkok, Thailand.

13. Thomas L. Saaty, 'Elements of Queuing Theory with Application,' McGRAW-HILL Book Company, New York, Toronto, London, chapter-5, 1981

14. Paul G. Hoest, Sidney C. Port, Charles I. Stone, 'Introduction to Stochastic Processes,' chapter-3, Universal Book Stall New Delhi, 2002

15. Anastrasi G., D. Grillo, L. Lenzini and E. Mingozzi, "A bandwidth reservation protocol for speech/data integration in TDMA-based advanced mobile systems," in Pro. *IEEE INFOCOM'96*, San-Francisco, CA, 1996

16. Zhang M., 1989 "Comparisons of channel assignment strategies in cellular mobile telephone systems," *IEEE Transaction on Vehicular Technology*, **138**, **6**, 211-215.

17. Bin Li, Lizhong Li, Bo Li, Krishna M. Sivalingam and Xi-Ren, 2004 "Call admission control for vioce/data integrated cellular networks: performance analysis and comparative study" *IEEE Journal on Selected Areas Communications*, **44**, **4**, 718.

18. Sampath A. and J. M. Holtzman, 1997 "Access control of data in integrated voice/data CDMA systems: benefits and tradeoffs," *IEEE journal on Selected Areas in Communications*, **15**, **8**, 1511-1526.

19. Bing Zheng and Mohammed Atiquzzaman, 1999 "Traffic Management of Multimedia over ATM Networks," *IEEE Communications Magazine*, 33-38.

20. Bing Zheng and M. Atiquzzaman, 2003 "A novel scheme for streaming multimedia to personal wireless hand held device," *IEEE Transactions on Consumer Electronics*, **49**, **1**, 32-40.

21. Sang H. Kang, Yong Han Kim, Dan K. Sung and Bong D. Choi, 2002 "An application of Markovian Arrival Process (MAP) to modeling superposed ATM cell stream," IEEE Transaction on Communications, **50**, **4**, 633-642.

22. Dianati M., Xinhua Ling, K. Naik and Xuemin Shen, 2006 "A node-cooperative ARQ scheme for wireless Ad-Hoc network," IEEE Transaction on Vehicular Technology, 55, 3, 1031-1044.