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# Sensitivity Analysis of Physiographic Parameters

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**Abstract**

The surface hydrologic behavior of a small natural hilly watershed of bridge no.319 of the Ministry of Railways, Government of India has been studied in details by the application of 'Kinematics Wave (KW) Theory' to find out the sensitivity of physiographic parameters overland slope, overland roughness, channel slope and channel roughness of overland and channel flow. The overland slope and overland roughness is more sensitive than channel slope and channel roughness for overland and channel flow.

**List of Symbols**

| | |
|---|---|
| A | Area of watershed, cross-sectional area |
| $A_0$ | Area of overland flow element |
| $\bar{A}$ | Average of the whole representative area |
| n | Manning roughness of overland |
| $S_0$ | Overland slope/bed slope |
| $n_1$ | Manning roughness of channel flow |
| $S_{01}$ | Channel slope |
| P | Pressure |
| q | Lateral inflow per unit length |
| $R_0$ | Hydraulic radius |
| $S_f$ | Frictional slope |
| u | x-component of mean velocity |
| v | y-component of mean velocity |
| w | z-component of mean velocity |
| X | Body force along x-direction |
| Y | Body force along y-direction |
| Z | Body force along z-direction |
| $\mu$ | Viscosity |
| $\rho$ | Mass density |
| $\alpha_0$ | Kinematic wave routing parameter of overland flow |
| $\alpha_k$ | Kinematic wave routing parameter of channel flow |
| $\Delta x$ | Space step |
| $\Delta t$ | Time step |
| $\phi$ | Infiltration index |
| K.W | Kinematic wave |

## I. Introduction

In the recent past, in many developing countries a good deal of research has been carried to solve the problems of 'large basins' whereas not much has been done with regard to the hydrologic problems of small watersheds. Small watersheds play important roles e.g. a village pond catered by its own small watershed; in hilly watersheds, the generated runoff causes flash flood, resulting into disruptions of communication lines etc. Therefore, it is necessary to look into these aspects of the hydrologic problems with greater attention. The hydrologic response of a small watershed depends upon the mechanics of surface runoff, which is primarily a nonlinear process. In this work, the surface hydrologic behavior of a small natural hilly watershed has been studied in details by the application of 'Kinematics Wave (KW) Theory' to find out the sensitivity of physiographic parameters of overland and channel flow. The 'Kinematic Wave (KW) Theory' has been preferred because this uses the simpler form of De Saint Venant[1] wave models. Stephenson and Meadows[2] found that the KW models perform better as these are based on the physics of the runoff process, most significantly, these models do account for the nonlinearity of the process without unduly complicating the solution procedures, which became unmanageable. Further, for both the components i.e. the overland and the channel flow routing, the KW models have been found to be accurate and efficient by Overton and Meadows[3]. Hossain[4] found that the KW theory is a powerful tool in computing the surface hydrologic responses of small watersheds of tropical region.

## II. Objective of the Study

The main objective of the present work may thus be summarized as to carry out the sensitivity analysis for the physiographic parameters of overland as well as cannel flow elements and to identify the parameters which are most sensitive.

## III. Kinematic Wave Equations from Saint Venant Equations

The equation of continuity and the equation of momentum for gradually varied, unsteady, one dimensional, incompressible flows were developed by French mathematician De Saint Venant[1]. These two equations are quasilinear, hyperbolic, partial differential equations. In its one dimensional form, these equations describe the changes in stream flow in the vertical and in the longitudinal directions.

The St. Venant equations characterizing the dynamic flow can be written as:

Continuity:
$$\frac{\partial A}{\partial t}+\frac{\partial Q}{\partial x}=q+(i-\phi) \quad \cdots (1)$$

Momentum:
$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+g\frac{\partial y_0}{\partial x}=g(s_f-s_0)-q\left(\frac{u-v}{A}\right)\cdots (2)$$

The equation (2) may be rewritten in the following form for a ready reference to the various types of wave models that are recognized.

Equation of motion:
$$\underbrace{\frac{1}{g}\frac{\partial u}{\partial x}}_{\mathrm{I}}+\underbrace{\frac{u}{g}\frac{\partial u}{\partial x}}_{\mathrm{II}}+\underbrace{\frac{\partial y_0}{\partial x}}_{\mathrm{III}}+\underbrace{(s_f-s_0)}_{\mathrm{IV}}=0\cdots(3)$$

Term: I II III IV

Wave model and terms used are:

(I) Kinematics wave only term $IV = 0$

(II) Dynamic wave $I + II + III + IV = 0$

and other terms are neglected.

By making suitable assumptions the KW equations can be deduced from the dynamic equations (Equations 1 and 2) given by De Saint Venant and these can be applied to channel flows.

In the present situation, the derivatives of the energy and velocity terms in the momentum equation (2) are very small in comparison with gravity and frictional forces. This allows assuming that the bed slope is approximately equal to the friction slope (i.e. $S_0 = S_f$). Under the above conditions and if there is no appreciable as the function of area of flow only and can be written as $Q = \alpha A^{m} \cdots (4)$

where $\alpha$ and m are known as kinematics wave routing parameters which are directly related to the watershed and flow characteristics.

**IV. Elements used in Kinematics Wave Models**

In this work, for computational purpose, the following two types of elements have been identified:

(i) Overland flow elements and

(ii) Channel flow elements (Fig. 1)

Overview of simplified arrangement of the above mentioned two elements, on reference to a small watershed is shown in Fig. 1(a), (b), (c), (d).

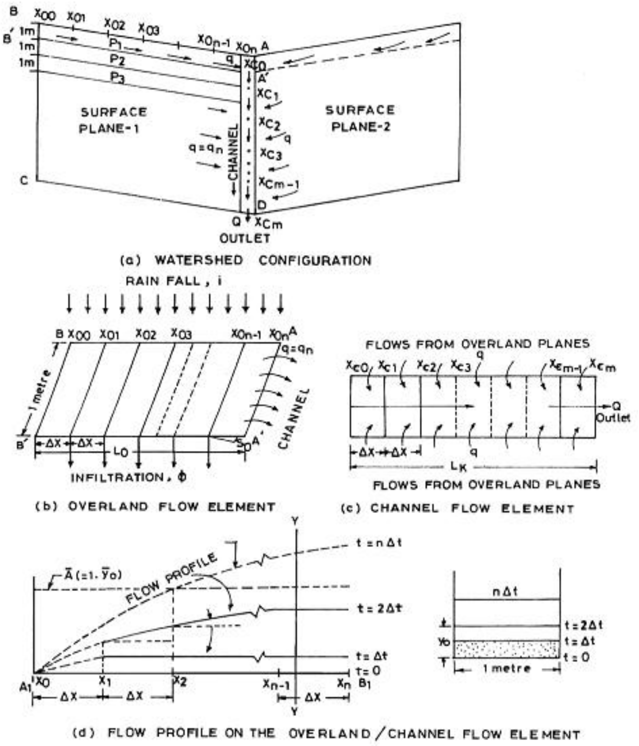



**Fig.1.** Generation of flow profile (after Hossain, MM, 1989)

**V. Kinematic Wave Equation for the Overland Flows**

It is assumed that the overland flow is in the forms of sheet flow. A unit width of the plane has been considered for the computational aspects of the runoff generation. The KW equations can, therefore, be written as:

$$\frac{\partial y_0}{\partial t} + \frac{\partial q}{\partial x} = i_e = i - \Phi \cdots (5)$$

$$q = \alpha_0 y_0^{m_0} \cdots (6)$$

where $\alpha_0$ and $m_0$ are kinematics wave routing parameters which are directly related to conveyance of particular surface (i.e. to the slope and its roughness), q is discharge per unit width of overland flow, $y_0$ is the mean depth and $i_e$ is the rainfall excess intensity [precipitant (i) – infiltration ( $\Phi$ )].

**VI. Determination of Kinematic Wave Routing Parameters $\alpha_0$ and $m_0$ for Overland Flows**

The overland flow on a wide plane has been considered as a sheet flow. Therefore, the kinematics wave equations for the overland flow segments have been derived from equation (6) along with the Manning's equation. The steady state Manning's equation for discharge per unit width (q) can be written as:

$$q = \frac{1}{n} A_0 R_0^{\frac{2}{3}} S_0^{\frac{1}{2}} \cdots (7)$$

where:

$n =$ Manning's roughness coefficient of overland flow

$A_0 =$ area of overland flow element

$R_0 =$ the hydraulic radius, and

$S_0 =$ overland slope.

For sheet flow on a plane of unit width, $A_0$ and $R_0$ can be replaced by $y_0$ i.e. the mean depth of flow (since $A_0 = 1. y_0$ and $R_0 = y_0$).

Substituting their values in equation (7) the discharge per unit width (q) can be written as:

$$q = \frac{1}{n} y_0^{\frac{5}{3}} S_0^{\frac{1}{2}} \cdots (8)$$

Comparing of equations (6) and (8) reveals that

$$\alpha_0 = \frac{1}{n} S_0^{\frac{1}{2}} \cdots (9)$$

and $$m_0 = \frac{5}{3} \cdots (10)$$

If the parameters $n$ and $S_0$ being known, then values of $\alpha_0$ is worked out from equation (9)

**VII. The Final Form of Kinematic Wave equations for the Overland Flows.**

Combing equations (5) & (6) and substituting $m_0$ given in equation (10), the following complete form kinematics wave equation for overland flow is derived as:

$$\frac{\partial y_0}{\partial t}+\frac{5}{3}\frac{\partial(\alpha_0\, y_0^{\frac{2}{3}})}{\partial x}=i_e \quad \cdots (11)$$

In equation (11), $y_0$ is only dependent variable which is a function of x, t and rainfall excess intensity ($i_e$). Thus $y_0$ can be determined explicitly by using equation (11). From computing values of $y_0$, the overland surface runoff per unit width can be computed by using equation (5)

**VIII. Kinematics Wave Equations for the Channel Flows**

For the channel flows, the kinematics wave equations can be written as follows:

$$\frac{\partial A}{\partial t}+\frac{\partial Q}{\partial x}=q \quad \cdots (12)$$

$$Q=\alpha_k A^{m_k} \quad \cdots (13)$$

where A= cross sectional area of the channel;
q = lateral inflows per unit length of the channel;
$\alpha_k$ & $m_k$ are the kinematics wave routing parameters which are directly related to the watershed and the channel flow characteristics.

In order to apply the equations (12) & (13), it is necessary to know the values of the parameters $\alpha_k$ and $m_k$ for the known channel physiography.

**IX. Trapezoidal Channel Cross Section**

A trapezoidal cross-section is the most general type of channel cross-section. The Manning's equation for discharge (Q) in a channel is given by

$$Q=\frac{1}{n_1}S_{01}^{\frac{1}{2}}AR^{\frac{2}{3}} \quad \cdots (14)$$

For the trapezoidal channel cross-section, equation (14) can be written as below:

$$Q=\frac{S_{01}^{\frac{1}{2}}}{n_1}A_c^{\frac{5}{3}}\left[\frac{1}{B+2y_c\sqrt{(1+Z^2)}}\right]^{\frac{2}{3}} \quad \cdots (15)$$

where $A_c$ is the area of the effective cannel cross-section at depth $y_c$, B is the channel bottom width, Z is the channel side slope.

**X. The Final Form of Kinematic Wave Equations for the Channel Flows**

The unknown parameters for the channel shapes under consideration i.e. $\alpha_k$ and $m_k$ being the unknown functions. The KW equation for the channel flow can be written by combining equations (12) and (13) as given below:

$$\frac{\partial A}{\partial t}+\frac{\partial(\alpha_k A^{m_k})}{\partial x}=q \quad \cdots (16)$$

If $\alpha_k$ is independent of x, then the equation (16) becomes:

$$\frac{\partial A}{\partial t}+\alpha_k m_k\frac{\partial(A^{m_k-1})}{\partial x}=q \quad \cdots (17)$$

Equation (17) is thus considered the final form of kinematics wave equation for channel flows. In this equation $A^{m_k}$ is only the dependent variable whereas $m_k$, t and q are independent variables. If $A^{m_k}$ is found out from equation (17) and is substituted in equation (13), the discharge (Q) from the channel is computed.

**XI. Different Scheme for the Solution of Kinematic Wave Equation**

The three different kinds of computational schemes as brief is shown in Fig. 2 have been used in this work for the solution of KW equations when applied to different watersheds under investigation.
Forward in time and backward in space, Scheme I
Backward in time and forward-in-space, Scheme II
Backward-in-time and backward in-space, Scheme-III

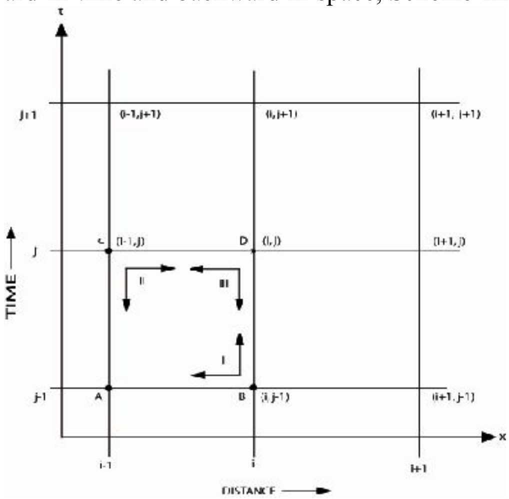


**Fig.2.** Space-time discretization for the computational schemes I, II & III (after Hossain, MM, 1989)

**XII. Computation Scheme I for the Solution of Kinematic Wave Equations**

As mentioned and shown in Fig.2. the Scheme I is defined as forward-in-time and backward-in-space. For this scheme, the temporal derivative of cross sectional area is evaluated between two points B and D in Fig.2 at the grid point (i, j-1) and thus can be written as

$$\frac{\partial A}{\partial t}\approx\frac{\Delta A}{\Delta t}=\frac{A_{i,j}-A_{i,j-1}}{\Delta t} \quad \cdots (18)$$

And the spatial derivative of area is evaluated between points A and B at the grid point(i, j-1) and gives

$$\frac{\partial A}{\partial x}\approx\frac{\Delta A}{\Delta x}=\frac{A_{i,j-1}-A_{i-1,j-1}}{\Delta t} \quad \cdots (19)$$

The average area is taken between two points A and B and gives

$$\bar{A} \approx \frac{A_{i,j-1} + A_{i-1,j-1}}{2} \quad \cdots \quad (20)$$

By making use of equation (18), (19) and (20) the general KW equation $\frac{\partial A}{\partial t} + \alpha_m A^{m-1} \frac{\partial A}{\partial x} = q$ can be written in terms of cross sectional area in its complete finite difference forms at the grid point (i,j-1) as below:

$$\frac{A_{i,j} - A_{i,j-1}}{\Delta t} + \alpha_m \left[\frac{A_{i,j-1} + A_{i-1,j-1}}{2}\right]^{m-1} \left[\frac{A_{i,j-1} - A_{i-1,j-1}}{2}\right] = \bar{q} \quad \text{...} \ (21)$$

In this study: $\bar{q} = q \quad \cdots \quad (22)$

The values of A are known at the grid points (i-1, j-1) and (i, j-1). Therefore in equation (21), the only unknown is $A_{i,j}$ and its value may be computed for the adopted values of $\alpha$ and m i.e. for the overland and channel flow, as the case may be.

Therefore, equation (21) can also be written in terms of $A_{i,j}$ in the following form explicitly:

$$A_{i,j} = \bar{q}\Delta t + A_{i,j-1} - \alpha m \frac{\Delta t}{\Delta x} \left[\frac{A_{i,j-1} + A_{i-1,j-1}}{2}\right]^{m-1} \left[A_{i,j-1} - A_{i-1,j-1}\right] \quad --- \ (23)$$

If $A_{i,j}$ is known from equation (23), then the corresponding $Q_{i,j}$ can be computed from equation (4) and is given by

$$Q_{i,j} = \alpha (A_{i,j})^m \quad \cdots \quad (24)$$

This is a direct method of computing the cross-sectional areas and the corresponding discharges. It is applied to the overland as well as the channel flow components with minor modifications in the relations of equations (23) and (24).

## XIII. Computational Scheme II for the Solution of Kinematic Wave Equations

It is seen from Fig.2. that the Scheme II is defined as backward-in-time and forward- in-space in its finite difference forms. In this scheme, the spatial derivative of discharge is evaluated between two points C and D, at the grid point (i-1, j) and can be written as:

$$\frac{\partial Q}{\partial x} \approx \frac{\Delta Q}{\Delta x} = \frac{Q_{i,j} - Q_{i-1,j}}{\Delta x} \quad \cdots \quad (25)$$

And the temporal derivative of area of cross section is computed between points C and A at the grid point (i-1, j) as:

$$\frac{\partial A}{\partial t} \approx \frac{\Delta A}{\Delta t} = \frac{A_{i-1,j} - A_{i-1,j-1}}{\Delta t} \quad \cdots \quad (26)$$

Substituting the above values in the kinematic wave equation (12), the following equation is obtained.

$$\frac{Q_{i,j} - Q_{i-1,j}}{\Delta x} + \frac{A_{i-1,j} - A_{i-1,j-1}}{\Delta t} = \bar{q} \quad \cdots \quad (27)$$

In equation (27) all the values are known except $Q_{i,j}$. Therefore this equation can be written in terms of $Q_{i,j}$ as:

$$Q_{i,j} = Q_{i-1,j} + \bar{q}\Delta x - \frac{\Delta x}{\Delta t}\left[A_{i-1,j} - A_{i-1,j-1}\right] \quad \cdots \quad (28)$$

Computing the value of $Q_{i,j}$ from equation (28) and substituting in equation (24), the area of cross section at the point (i, j) is to be determined by

$$A_{i,j} = \left(\frac{Q_{i,j}}{\alpha}\right)^{\frac{1}{m}} \quad \cdots \quad (29)$$

## XIV. Watershed of our Study

In developing countries, there is a limitation for collection and computation of detailed hydrological data due to financial constraints. Non-availability of equipments, technical personnel and other facilities also adversely affect this programme. The available data has been taken from the PhD thesis of Hossain[4] (1989). The flash floods so produced pose dangers to the numerous culverts and bridges of the railways which are located in this region. A small watershed was identified where short duration rainfall and runoff data of major storm events were collected for in-depth study of the mechanics of the runoff generation. This bridge was identified as Bridge No. 319.

## XV. Physiographic Details of Watershed

The small watershed of Railway Bridge No. 319 selected for this study is situated in Bangalore district of Karnataka of India. This bridge is located on Arsikere Bangalore Section of the Indian Railways. The index maps giving details of the watershed is given in Fig.3.

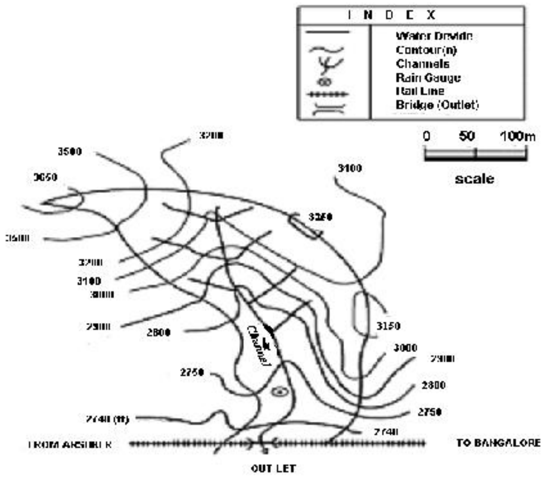
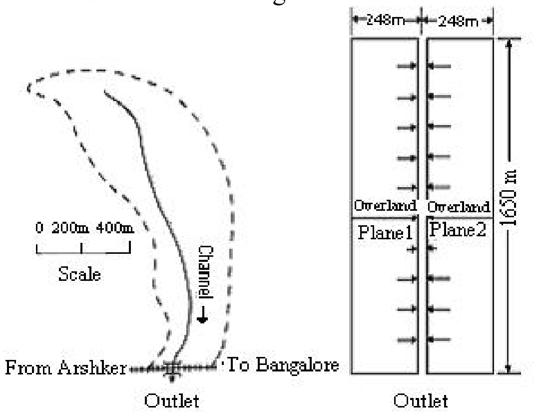



**Fig.3.** The index map of bridge no.319 (after Hossain, MM, 1989)

## XVI. Estimation of Physiographic Parameters for Bridge No. 319

The topographic details of this watershed are shown in Fig. 3. In this natural watershed, only one main drainage channel exists that is in the central part of the watershed. The lumped physiographic model is used to compute the model parameters for the application of KW theory. For this purpose, a lumped model of the type is adopted for the estimation of parameters.

The equivalent watershed has been obtained by dividing equally the total drainage area into the two sides 1650 meters long main channel. The schematic representation of this model is shown in Fig. 4.




(a) Watershed area (b) lumped system presentation

**Fig.4.** Schematic presentation through a lumped model (after Hossain, MM, 1989)

## XVII. The Sensitivity Analysis

In section XV theoretical aspects of sensitivity analysis of model parameters were discussed. Following the method of perturbation, the sensitivity analysis was conducted with respect to the following physiographic parameters.

(a) In overland flow phase (i) Overland slope

(ii) Overland roughness

(b) In channel flow phase (i) Overland slope

(ii) Overland roughness (iii) Channel roughness

(iv) Channel side slope.

To compute the response through the application of KW models, the computational scheme (I & II) is preferred. It is because stability and convergence of the scheme remain more or less ensured through a proper selection of smaller values of time and space steps.

It is proposed to conduct this analysis with a time step.

## XVIII. Overland Flow of Watershed for different values of Overland Roughness (n)

In overland there are two physiographic parameters land roughness and slope. The effect of overland roughness on overland flow of watershed is shown in the following Fig. 5.

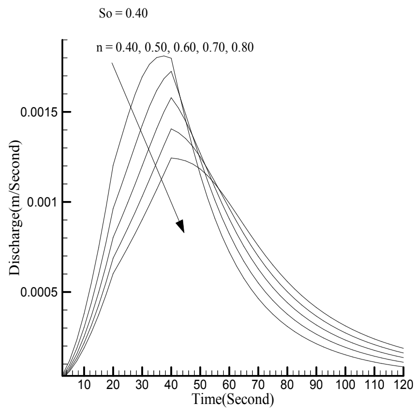



**Fig. 5.** Overland flow for different values of overland roughness.

From the above figure we observed that if the overland roughness is increasing then peak flow is decreasing. So the overland roughness (n) is sensitive.

## XIX. Overland Flow of Watershed for Different Values of Overland Slope (So)

Different overland has different overland slope. It has a effect on overland flow. The effect of overland slope on overland flow of watershed is shown in the following Fig.6.

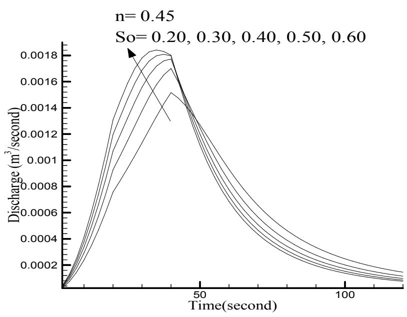



**Fig. 6.** Overland flow for different values of overland slope.

From the above figure we observe that if the overland slope is increasing then the peak flow is increasing for high slope. So the overland slope is sensitive.

## XX. Channel Flow of Watershed for Different Values of Overland Roughness (n)

Flow of watershed is made of overland and channel flow. Overland roughness is a important factor for channel flow. The effect of overland roughness on channel flow of watershed is shown in the Fig. 7.

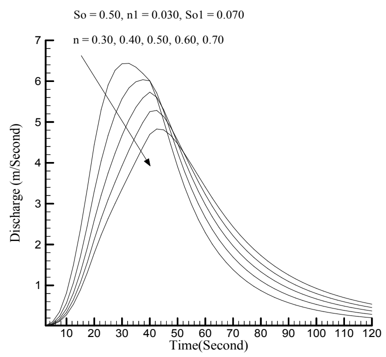



**Fig. 7.** Channel flow routing for different value of overland roughness (n)

From Fig.7, we observed that if overland roughness is increasing then the peak flow is decreasing and the flow routing changes in shapes quickly. So the overland roughness on channel flow is sensitive.

## XXI. Channel Flow of Watershed for Different Values of Channel Roughness (n1)

Different channel has different channel roughness. It is a factor of channel flow. The effect of channel roughness on channel flow of watershed is shown in the following Fig.8.

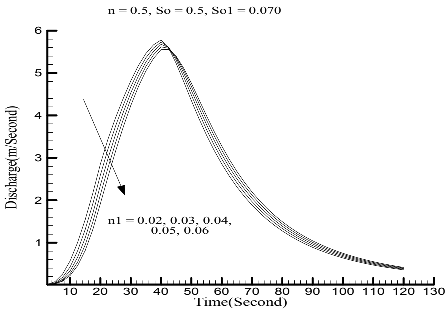



**Fig. 8.** Channel flow routing for different value of channel roughness (n1)

From the above figure it appears easily observed that the effect of channel roughness on channel flow peak is sensitive but is not like overland roughness. So channel roughness is sensitive but not like as overland roughness on channel flow.

## XXII. Channel Flow of Watershed of Bridge No.319 for Different Value of Overland Slope (So)

Channel flows depend on different physiographic parameters of overland and channel. Here the effect of overland slope (So) on channel flow of watershed is shown in the following Fig.9.

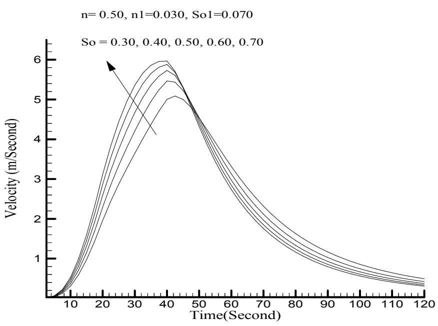



**Fig. 9.** Channel flow routing for different value of overland slope.

From the above figure, we observe that the effect of overland slope on the peak flow routing of channel flow is increasing then flow routing is increasing, but after a period, the flow is not increasing. So overland slope is not so sensitive as overland roughness for channel flow.

## XXIII. Channel Flow Routing of Watershed for Different Values of Channel Slope (So1)

Different channel has different channel slope. It is a factor of channel flow. The effect of channel slope on channel flow of watershed is shown in the following Fig.10.

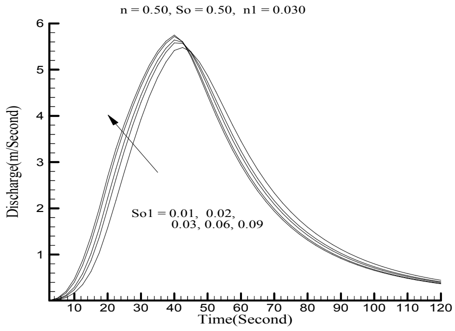



**Fig. 10.** Channel flow routing for different values of channel slope.

From the above figure, it may be noticed that the effect of channel slope on channel flow peak is not as sensitive as overland slope

## XXIV. Discussion and Conclusion

The three different cross-section of a natural channel are Trapezoidal, Rectangular and Triangular. Most of the natural channels nearly conform to trapezoidal shapes. The other two cross-sections are particular cases of the trapezoidal section under certain conditions.

We have discussed the channel flow routing and overland flow routing of watershed and analysised the sensitivity of physiographic parameters. From the discussion we observed that the flow routing depend on different kind of channel and overland flow parameters.

From the above discussion we come in conclusion that the overland roughness and overland slope are more sensitive than the other parameters on overland and channel flow.

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