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Solutions of Fuzzy Linear Systems

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Abstract

In this paper, we investigate the solution of system of fuzzy linear equations, $Ax = b$, where A is a crisp real $n \times n$ matrix and b is a vector consisting of n triangular fuzzy numbers. Assuming that the unknown vector x is a fuzzy number vector of the same type as b, and defining the addition and scalar-multiplication by Zadeh's extension principle, we propose a method of solution that replaces the $n \times n$ fuzzy system by a $3n \times 3n$ crisp linear system $MX = B$, where M is a block symmetric matrix depending on A and B is a vector whose components are rearranged parameters of the fuzzy numbers in b. We provide conditions for the existence of a unique fuzzy solution of the system under investigation. We use the *Mathematica* package for symbolic and numeric computation.

***Keywords*:** System of Fuzzy Linear Equations, Fuzzy Linear Equations, Triangular Fuzzy Numbers, Block matrix, Block Symmetric Matrix, Determinant of Block Matrix, Inverse of Block Matrix.

I. Introduction

In application of mathematics to problems in sciences and engineering, one often encounters systems of linear equations. Various methods have been developed to solve such systems analytically or numerically. Since in many applications, some parameters need to be represented by fuzzy numbers rather than crisp numbers, it is important that we address the problem of solving linear systems $Ax = b$, where some or all components of the matrix A and the vector b are fuzzy numbers. Such a system of fuzzy linear equations (SFLE, in short) has been investigated by a host of researchers, including Friedman[1], Peeva[2], Zhang[3].

In this paper, a particular method for solving a SFLE $Ax = b$ is presented, where $A = (a_{ii})$, $1 \le i, j \le n$ is a crisp $n \times n$ matrix and b is a vector of triangular fuzzy numbers (TrFN), assuming the unknown vector x is also a TrFN. First of all we transform the $n \times n$ fuzzy system $Ax = b$ to a $3n \times 3n$ crisp system $MX = B$, where M is a block symmetric matrix constructed from A, and B is a vector of rearranged parameters of the fuzzy numbers in b. We define the conditions for existence of a unique fuzzy solution and then solve the expanded system numerically. An algorithm is given to solve the system and we also develop a programming code in *Mathematica*. For *Mathematica* we refer to [Don[4], Wolfram[5]]. Finally the validity of the algorithm and program is illustrated by solving some examples.

II. Preliminaries

L.A. Zadeh[6] first introduced the concept of fuzzy sets, and today fuzzy mathematical literature abounds with studies on fuzzy numbers and their applications. For characterizations of fuzzy numbers and their arithmetic operations, we refer to [Congxin and Ma[7,8], Dubois and Prade[9], Goetschel and Voxman[10], Kaufman and Gupta[11], Klir and Yuan[12]].

In [12], a fuzzy number is defined as follows:

Definition 2.1 (Fuzzy Number)

A fuzzy set $\mu : \mathbf{R} \to I = [0,1]$ is said to be a fuzzy number if it possesses the following properties

1. μ is a normal fuzzy set ;
2. for every $\alpha \in (0,1]$, the α-cut, μ^{α} is a closed interval denoted by $[\underline{\mu}(\alpha), \overline{\mu}(\alpha)]$;
3. the support of μ, μ^{0+}, is bounded.

We shall denote the set of all fuzzy numbers by ***FN*** (**R**).

In this paper, we shall be using triangular fuzzy numbers, as such numbers are most commonly used.

Definition 2.2 (Triangular Fuzzy Number)

A fuzzy number $\mu \in$ ***FN*** (**R**) is said to be a triangular fuzzy number (TrFN, in short) if there exists three real numbers p, q and s such that $p \le q \le s$, and

1. $\mu(x) = 1$, for x = q;
2. for every $\alpha \in I_0 = (0,1]$,

$$\mu^{\alpha} = [\underline{\mu}(\alpha), \overline{\mu}(\alpha)] = [p + (q - p)\alpha, s - (s - q)\alpha];$$

3. $\mu^{0+} = (p, s)$.

Such a fuzzy number will be denoted by $Tr[p, q, s]$, and the set of all triangular fuzzy numbers will be denoted by

TrFN(**R**). $[p,q,s]$ is called the parameter-vector of $Tr[p,q,s]$. In case $p<q<s$, the membership function of $Tr[p,q,s]$ is

$$\mu(x)=\begin{cases}0 & , \text{ if } \ x<p \\ (x-p)/(q-p) & , \text{ if } \ p\le x<q \\ 1 & , \text{ if } \ x=q \\ (s-x)/(s-q) & , \text{ if } \ q<x\le s \\ 0 & , \text{ if } \ x>s\end{cases}$$

For such functions, we have:

Proposition 2.1 [11]

If $\mu=Tr[p,q,s]$, $\gamma=Tr[k,l,m]$ and c is a real number, then

I. $\mu=\gamma$ if and only if $p=k$, $q=l$ and $s=m$.

II. $\mu+\gamma=Tr[p+k,q+l,s+m]$.

III. $c\mu=\begin{cases}Tr[cp,cq,cs] \ ,\text{if } \ c\ge 0 \\ Tr[cs,cq,cp] \ ,\text{if } \ c<0.\end{cases}$

III. The Model and Solution

Consider the system of fuzzy linear equations (SFLE, in short)

$$\left.\begin{aligned} a_{11}x_1+a_{12}x_2+\cdots\cdots\cdots+a_{1n}x_n&=b_1 \\ a_{21}x_1+a_{22}x_2+\cdots\cdots\cdots+a_{2n}x_n&=b_2 \\ \vdots \\ a_{n1}x_1+a_{n2}x_2+\cdots\cdots\cdots+a_{nn}x_n&=b_n\end{aligned}\right\} \tag{3.1}$$

in "unknowns" $x_1,x_2,\cdots,x_n$, where the coefficients $a_{ij}\in$ **R** for $1\le i,j\le n$ and $b_i=Tr[p_i,q_i,s_i]$ ∈ *TrFN*(**R**) for $1\le i\le n$. In matrix-vector form (3.1) can be expressed as

$$Ax=b, \tag{3.2}$$

where $A=(a_{ij})$, $1\le i,j\le n$, is a crisp $n\times n$ matrix and $b=(b_1,b_2,\cdots,b_n)^t$ is a column vector of TrFN's, $b_i=Tr[p_i,q_i,s_i]$, $1\le i\le n$ and $x=(x_1,x_2,\cdots,x_n)^t$ is an unknown column vector.

Definition 3.1

A vector $x=(x_1,x_2,\cdots,x_n)^t$ ∈ *TrFN*(**R**)^n, where $x_i=Tr[u_i,v_i,w_i]$, $1\le i\le n$, is called a *solution* of the SFLE (3.2) if

$$\sum_{j=1}^{n}a_{ij}Tr[u_j,v_j,w_j]=Tr[p_i,q_i,s_i],\ 1\le i\le n. \tag{3.3}$$

The system (3.2) does not always have a solution, as we shall see.

IV. The Solution Procedure

Suppose that system (3.2) has a solution $x=(x_1,x_2,\cdots,x_n)^t$ ∈ *TrFN*(**R**)^n, where $x_i=Tr[u_i,v_i,w_i]$, $1\le i\le n$. Then the *i*-th equation of the system (3.2) is

$$\begin{aligned} a_{i1}Tr[u_1,v_1,w_1]+\cdots+a_{ii}Tr[u_i,v_i,w_i]+\cdots+ \\ a_{in}Tr[u_n,v_n,w_n]=Tr[p_i,q_i,s_i]\end{aligned} \tag{4.1}$$

If $a_{ij}\ge 0$, $1\le j\le n$, then this simplifies to

$$Tr\left[\sum_{j=1}^{n}a_{ij}u_j,\sum_{j=1}^{n}a_{ij}v_j,\sum_{j=1}^{n}a_{ij}w_j\right]=Tr[p_i,q_i,s_i] \tag{4.2}$$

and we get three crisp equations

$$\sum_{j=1}^{n}a_{ij}u_j=p_i,\ \sum_{j=1}^{n}a_{ij}v_j=q_i,\ \sum_{j=1}^{n}a_{ij}w_j=s_i. \tag{4.3}$$

But if any $a_{ij}<0$, the above simplification is not possible

However, the SFLE (3.2) can be expanded to a $3n\times 3n$ crisp system of linear equations by defining m_{ij} for $1\le i,j\le 3n$ as follows:

for k = 0 and 1 and $1\le i,j\le n$,

$$m_{2kn+i,2kn+j}=\begin{cases}a_{ij} & \text{if } \ a_{ij}>0 \\ 0 & \text{if } \ a_{ij}\le 0\end{cases}, \tag{4.4}$$

$$m_{2kn+i,2(1-k)n+j}=\begin{cases}a_{ij} & \text{if } \ a_{ij}<0 \\ 0 & \text{if } \ a_{ij}\ge 0\end{cases}, \tag{4.5}$$

$$m_{n+i,n+j}=a_{ij} \quad \text{and}$$
$$m_{i,n+j}=m_{n+i,j}=m_{n+i,2n+j}=m_{2n+i,n+j}=0 \tag{4.6}$$

The expanded crisp system is

$$\begin{gathered} m_{11}u_1+\cdots+m_{1n}u_n+m_{1,n+1}v_1+\cdots+m_{1,2n}v_n+m_{1,2n+1}w_1+\cdots+m_{1,3n}w_n=p_1 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ m_{n1}u_1+\cdots+m_{nn}u_n+m_{n,n+1}v_1+\cdots+m_{n,2n}v_n+m_{n,2n+1}w_1+\cdots+m_{n,3n}w_n=p_n\end{gathered}$$

$$m_{n+1,1}u_1+\cdots+m_{n+1,n}u_n+m_{n+1,n+1}v_1+\cdots+m_{n+1,2n}v_n+m_{n+1,2n+1}w_1+\cdots+m_{n+1,3n}w_n=q_1$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \qquad (4.7)$$
$$m_{2n,1}u_1+\cdots+m_{2n,n}u_n+m_{2n,n+1}v_1+\cdots+m_{2n,2n}v_n+m_{2n,2n+1}w_1+\cdots+m_{2n,3n}w_n=q_n$$
$$m_{2n+1,1}u_1+\cdots+m_{2n+1,n}u_n+m_{2n+1,n+1}v_1+\cdots+m_{2n+1,2n}v_n+m_{2n+1,2n+1}w_1+\cdots+m_{2n+1,3n}w_n=s_1$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$
$$m_{3n,1}u_1+\cdots+m_{3n,n}u_n+m_{3n,n+1}v_1+\cdots+m_{3n,2n}v_n+m_{3n,2n+1}w_1+\cdots+m_{3n,3n}w_n=s_n$$

which may be abbreviated as

$$MX=B, \qquad (4.8)$$

where $M=(m_{ij})$, $1\le i,j\le 3n$,

$$X=(u_1,u_2,\cdots,u_n,v_1,v_2,\cdots,v_n,w_1,w_2,\cdots,w_n)^t \qquad (4.9)$$

$$B=(p_1,p_2,\cdots,p_n,q_1,q_2,\cdots,q_n,s_1,s_2,\cdots s_n)^t$$

(4.10)

and the structure of M is

$$M=\begin{pmatrix} M_1 & O & M_2 \\ O & A & O \\ M_2 & O & M_1 \end{pmatrix} \qquad (4.11)$$

which is a block symmetric matrix of size $3n\times 3n$, where the block M_1 is the $n\times n$ matrix

$$M_1=(m_{ij}^{(1)}) \text{ with } m_{ij}^{(1)}=\begin{cases} a_{ij} & if\ a_{ij}>0 \\ 0 & if\ a_{ij}\le 0 \end{cases}, \qquad (4.12)$$

the block M_2 is the $n\times n$ matrix

$$M_2=(m_{ij}^{(2)}) \text{ with } m_{ij}^{(2)}=\begin{cases} a_{ij} & if\ a_{ij}<0 \\ 0 & if\ a_{ij}\ge 0 \end{cases}, \qquad (4.13)$$

the block $A=M_1+M_2$ and the block O is the $n\times n$ zero matrix.

Now, the crisp system (4.8) can be written as

$$\begin{pmatrix} M_1 & O & M_2 \\ O & A & O \\ M_2 & O & M_1 \end{pmatrix}\begin{pmatrix} u \\ v \\ w \end{pmatrix}=\begin{pmatrix} p \\ q \\ s \end{pmatrix}, \qquad (4.14)$$

where

$$u=(u_1,u_2,\cdots,u_n)^t,\ v=(v_1,v_2,\cdots,v_n)^t,$$
$$w=(w_1,w_2,\cdots,w_n)^t \qquad (4.15)$$
$$p=(p_1,p_2,\cdots,p_n)^t,\ q=(q_1,q_2,\cdots,q_n)^t,$$
$$s=(s_1,s_2,\cdots,s_n)^t \qquad (4.16)$$

The above construction is illustrated in the following example.

Example 1

Consider the 2×2 fuzzy linear system

$$x_1-2x_2 = Tr[-3,0,2]$$
$$3x_1+5x_2 = Tr[1,3,4]$$

If $x_1=Tr[u_1,v_1,w_1]$, $x_2=Tr[u_2,v_2,w_2]$ constitute a solution, then we obtain the following 6×6 system according to (4.7) applying (4.4), (4.5) and (4.6) .

$$\begin{array}{llllll} u_1 & & & & +(-2)w_2 & =-3 \\ 3u_1 & +5u_2 & & & & =1 \\ & & v_1+(-2)v_2 & & & =0 \\ & & 3v_1+5v_2 & & & =3 \\ & (-2)u_2 & & +w_1 & & =2 \\ & & & 3w_1 & +5w_2 & =4 \end{array}$$

Here $M_1=\begin{pmatrix} 1 & 0 \\ 3 & 5 \end{pmatrix}$ and $M_2=\begin{pmatrix} 0 & -2 \\ 0 & 0 \end{pmatrix}$.

V. Existence of Solution

The crisp system (4.8) has a unique solution if and only if the matrix M is nonsingular. Note that $A=M_1+M_2$, where M_1 and M_2 are defined in (4.12) and (4.13). Define

$$\overline{A}=M_1-M_2. \qquad (5.1)$$

Theorem 5.1

The matrix M is nonsingular if and only if the matrices A and $\overline{A}$ are both nonsingular.

Proof:

For any square blocks *A*, *B*, *C* and *D* of equal size, we have [14]

i). $\det(AB)=\det(A)\det(B)$.

ii). If $M=\begin{pmatrix} A & B \\ C & D \end{pmatrix}$, then

$$\det(M)=\det(AD-BC),$$

whenever at least one of the blocks *A*, *B*, *C* and *D* is equal to *O* (zero matrix).

By adding the $(2n+i)$-th row of M in (4.11) to its *i*-th row for $1\le i\le n$, we obtain

$$K_1=\begin{pmatrix} M_1+M_2 & O & M_1+M_2 \\ O & A & O \\ M_2 & O & M_1 \end{pmatrix},$$

and then subtracting the *j*-th column of K_1 from its $(2n+j)$-th column for $1\le j\le n$ we obtain

$$K_2 = \begin{pmatrix} M_1+M_2 & O & O \\ O & A & O \\ M_2 & O & M_1-M_2 \end{pmatrix}.$$

It follows that

$$\det(M) = \det(K_2)$$

$$= \det\left((M_1+M_2) \begin{pmatrix} A & O \\ O & M_1-M_2 \end{pmatrix} - (O \quad O) \begin{pmatrix} O \\ M_2 \end{pmatrix} \right)$$

$$= \det\left(A \begin{pmatrix} A & O \\ O & M_1-M_2 \end{pmatrix} \right)$$

$$= \det(A)\ \det(A)\ \det(M_1 - M_2)$$

$$= (\det(A))^2\ \det(M_1 - M_2)$$

$$= (\det(A))^2\ \det(\overline{A})$$

Thus $\det(M) = (\det(A))^2 \det(\overline{A})$.

Therefore $\det(M) \neq 0$ if and only if $\det(A) \neq 0$ and $\det(\overline{A}) \neq 0$.

This concludes the proof.

Theorem 5.2

If the matrix M, as defined in (4.11), is invertible, then

$$M^{-1} = \begin{pmatrix} G_1 & O & G_2 \\ O & A^{-1} & O \\ G_2 & O & G_1 \end{pmatrix}, \tag{5.2}$$

where

$$G_1 = \frac{1}{2}\left((M_1+M_2)^{-1} + (M_1-M_2)^{-1} \right) = \frac{1}{2}(A^{-1} + \overline{A}^{-1}) \tag{5.3}$$

$$G_2 = \frac{1}{2}\left((M_1+M_2)^{-1} - (M_1-M_2)^{-1} \right) = \frac{1}{2}(A^{-1} - \overline{A}^{-1}) \tag{5.4}$$

Proof:

Let M be an invertible matrix. Now in order to calculate M^{-1}, we observe that

$$\begin{pmatrix} M_1 & O & M_2 \\ O & A & O \\ M_2 & O & M_1 \end{pmatrix} \begin{pmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{33} \end{pmatrix} = \begin{pmatrix} I_n & O & O \\ O & I_n & O \\ O & O & I_n \end{pmatrix},$$

where I_n is an $n \times n$ identity matrix, and which leads to the following relations:

(i) $M_1G_{11} + M_2G_{31} = I_n$

(ii) $AG_{21} = O$

(iii) $M_2G_{11} + M_1G_{31} = O$

(iv) $M_1G_{12} + M_2G_{32} = O$

(v) $AG_{22} = I_n$

(vi) $M_2G_{12} + M_1G_{32} = O$

(vii) $M_1G_{13} + M_2G_{33} = O$

(viii) $AG_{23} = O$

(ix) $M_2G_{13} + M_1G_{33} = I_n$

Now from (i) and (iii) we get

$$G_{11} + G_{31} = (M_1 + M_2)^{-1} \quad \text{and}$$

$$G_{11} - G_{31} = (M_1 - M_2)^{-1}$$

which give

$$G_{11} = \frac{(M_1+M_2)^{-1} + (M_1-M_2)^{-1}}{2} = \frac{A^{-1} + \overline{A}^{-1}}{2}$$

and

$$G_{31} = \frac{(M_1+M_2)^{-1} - (M_1-M_2)^{-1}}{2} = \frac{A^{-1} - \overline{A}^{-1}}{2}.$$

Similarly from (vii) and (ix), we obtain

$$G_{33} = \frac{(M_1+M_2)^{-1} + (M_1-M_2)^{-1}}{2} = \frac{A^{-1} + \overline{A}^{-1}}{2} = G_{11}$$

and

$$G_{13} = \frac{(M_1+M_2)^{-1} - (M_1-M_2)^{-1}}{2} = \frac{A^{-1} - \overline{A}^{-1}}{2} = G_{31}.$$

Again considering (iv) and (vi), as $M_1 + M_2 = A \neq O$ and $M_1 - M_2 = \overline{A} \neq O$, we obtain

$$G_{12} + G_{32} = O \quad \text{and} \quad G_{12} - G_{32} = O$$

which give $G_{12} = G_{32} = O$.

Also from (ii), (v) and (viii), we get

$$G_{21} = O,\ G_{22} = A^{-1} \text{ and } G_{23} = O.$$

Now letting $G_{11} = G_{33} = G_1$ and $G_{13} = G_{31} = G_2$, the proof is completed.

Once M^{-1}, as defined in (5.2), is calculated by (5.3) and (5.4), we have, from (4.14)

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} G_1p + G_2s \\ A^{-1}q \\ G_2p + G_1s \end{pmatrix}. \tag{5.5}$$

Rearranging, we get

$$[u, v, w] = [G_1p + G_2s, A^{-1}q, G_2p + G_1s] \tag{5.6}$$

The *i*-th row of the right-hand side of (5.5) is a parameter vector of a trapezoidal fuzzy number if and only if

$$(G_1p + G_2s)_i \leq (A^{-1}q)_i \leq (G_2p + G_1s)_i \tag{5.7}$$

Thus we conclude

Theorem 5.3

The system $Ax = b$ in (3.2) has a unique solution $x = (x_1, x_2, \cdots, x_n)^t \in TrFN(\mathbf{R})^n$, where $x_i = Tr[u_i, v_i, w_i], 1 \leq i \leq n$, if and only if the following conditions hold:

i). A and $\overline{A}$ are both nonsingular, where $\overline{A}$ is as in (5.1);

ii). $G_1 p + G_2 s \le A^{-1} q \le G_2 p + G_1 s$,
where G_1, G_2 are as in (5.3) and (5.4), and
p, *q*, *s* are as in (4.16).

VI. Computation of Solution using MATHEMATICA

Summarizing the discussion in the previous section, we have the following algorithm for the calculating the solution of SFLE (3.2).

Step 1. Input the matrix A.

Step 2. Input *b* as a matrix whose rows are the parameter-vectors of the TrFNs.

Step 3. Compute det(*A*).
If $\det(A) = 0$, print "The method does not lead to a solution" and stop.
Else, continue.

Step 4. Compute M_1 and M_2 (as defined in (4.12-13)).

Step 5. Set $\overline{A} = M_1 - M_2$ and compute $\det(\overline{A})$.
If $\det(\overline{A}) = 0$, print "The method does not lead to a solution" and stop.
Else, continue.

Step 6. Compute A^{-1} and $\overline{A}^{-1}$.

Step 7. Set $G_1 = \frac{1}{2}(A^{-1} + \overline{A}^{-1})$ and
$G_2 = \frac{1}{2}(A^{-1} - \overline{A}^{-1})$.

Step 8. Set $p = col(1)$ of *b*, $q = col(2)$ of *b*
and $s = col(3)$ of *b*.

Step 9. Compute $u = G_1.p + G_2.s$, $v = A^{-1}.q$
and $w = G_2.p + G_1.s$.

Step 10. Check whether $u \le v \le w$.
If any of these inequalities is false, print, "The system does not have TrFN-vector solution", and end.
Else, print, "The system has unique TrFN-vector solution" and continue.

Step 11. Set the solution vector $x = (x_i)$, where
$x_i = (u_i, v_i, w_i)$, $1 \le i \le n$.

Step 12. The solution vector is $(Tr[x_i])_{1\le i\le n}$,
where x_i is the *i*-th component of the vector *x*.

We illustrate the above procedure with four examples using *Mathematica* code.

```
solution[A_, b_] := If[Det[A] == 0, Print["The method does not lead to a solution"],
            n = Dimensions[A][[1]];
            M1 := Table[If[A[[i, j]] ≥ 0, A[[i, j]], 0], {i, n}, {j, n}];
            M2 := Table[If[A[[i, j]] ≤ 0, A[[i, j]], 0], {i, n}, {j, n}];
            Ā := M1 - M2;
            If[Det[Ā] == 0, Print["The method does not lead to a solution"],
            G1 = (Inverse[A] + Inverse[Ā]) / 2;
            G2 = (Inverse[A] - Inverse[Ā]) / 2;
            p = Transpose[b][[1]]; q = Transpose[b][[2]]; s = Transpose[b][[3]];
            u = G1.p + G2.s; v = Inverse[A].q; w = G2.p + G1.s;
            test = Table[u[[i]] ≤ v[[i]] ≤ w[[i]], {i, n}];
            truth = Table[True, {i, n}];
            If[test ≠ truth, Print["The system does not have TrapFN-vector solution"],
            Print["The system has unique TrapFN-vector solution"];
            x = N[Table[{u[[i]], v[[i]], w[[i]]}, {i, n}]]]]];
```

Example 2 (*A* is singular)

```
Clear[A, b]
A = {{2, 0, 1}, {4, 2, 3}, {-5, 3, -1}};
b = {{3, 8, 11}, {-8, -4, 1}, {-2, 0, 5}};
solution[A, b]
The method does not lead to a solution
```

Example 3 ($\overline{A}$ is singular)

```
Clear[A, b]
A = {{1, -1}, {1, 1}};
b = {{-1, 3, 5}, {-4, 1, 8}};
solution[A, b]
The method does not lead to a solution
```

Example 4 (Solution fails)

```
Clear[A, b]
A = {{2, 3, 1}, {-4, 0, 3}, {1, 2, -1}};
b = {{-1, 2, 6}, {3, 5, 8}, {-2, 1, 3}};
solution[A, b]
The system does not have TrFN-vector solution
```

Example 5 (Solution exists)

```
Clear[A, b]
A = {{1, 2, -1, 0, 1}, {-1, 3, 2, 1, 0}, {0, -1, 3, 2, -1}, {2, 0, 1, -1, 3}, {1, -2, 0, 2, 1}};
b = {{2, 10, 17}, {-4, 9, 18}, {-13, -3, 8}, {-8, 0, 10}, {-2, 4, 13}};
solution[A, b]
The system has unique TrFN-vector solution
{{-1., 0., 2.}, {1., 3., 4.}, {-4., -2., 0.}, {3., 4., 5.}, {1., 2., 3.}}
```

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