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Determination of Optimal Smoothing Constants for Exponential Smoothing Method & Holt's Method

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Abstract

This paper concentrates on choosing the appropriate smoothing constants for Exponential Smoothing method and Holt's method. These two methods are very important quantitative techniques in forecasting. The accuracy of forecasting of these techniques depends on Exponential smoothing constants. So, choosing an appropriate value of Exponential smoothing constants is very crucial to minimize the error in forecasting. In this paper, we have showed how to choose optimal smoothing constants of these techniques for a particular problem. We have demonstrated the techniques by presenting a real life example and calculating corresponding forecast value of these two techniques for the optimal smoothing constants.

Keywords: Exponential Smoothing; Holt's Method; Smoothing Constants; Forecast Error

I. Introduction

Forecasting is a vital managerial activity as it is the first stage of every planning procedure. Many organizations have failed because of lack of forecasting or faulty forecasting on which the planning is based. Exponential Smoothing and Holt's method are two important forecasting techniques. These forecasting methods use constants that assign weights to current demands and previous forecasts to arrive at new forecasts. Their values influence the responsiveness of forecasts to actual demand and hence influence forecast error. Considerable effort has focused on finding the appropriate values to use. One approach is to use smoothing constants that minimize some functions of forecast error. Thus, in order to select the right constants for forecasting, different values are tried out on past time series, and the ones that minimize error functions like Mean Absolute Deviation (MAD), Mean Squared Error (MSE) and Mean Absolute Percentage Error (MAPE) used for forecasting accuracy.

In ordinary terms, an Exponential weighting scheme assigns the maximum weight to the most recent observation and the weight decline in a systematic manner as older and the older observations are included. Weight in the Exponential smoothing technique is given by Exponential smoothing constant (α).

Many authors used Exponential Smoothing method and Holt's Method in forecasting. James W. Taylor[1, 2] developed a new approach of smooth transition model by using Exponential smoothing. Sanjoy Kumar Paul[3] tried to determine the Exponential smoothing constant by minimizing the Mean Absolute Deviation (MAD) and Mean Square Error (MSE). P. Y. Lim and C. V. Nayar[4] used Single Exponential Smoothing method to forecast on solar irradiance and load demand. So, they tried to analysis the predictions for solar irradiance and load demand using two different Single Exponential Smoothing forecasting approaches. H. V. Ravinder[5,6] examined Exponential smoothing constants that minimize summary error associated with a large number of forecasts. Vikas Pratap Singh & Vivek Vijay[7] discussed the impact of trend and seasonality on 5-MW plant generation forecasting using Single Exponential smoothing method and forecasting values were computed for different values of smoothing factor (α).

When business organizations make forecast for a particular set of sales or demands, they have to use Exponential Smoothing methods and these methods use smoothing constants. So, forecast values are varied with the different values of the smoothing constants and forecasting error is also depended on these constants. Therefore, smoothing constants are central for successful forecasting with Exponential smoothing but there is no consistent guideline in the forecasting literature on how they should be selected. That's why we try to determine the suitable smoothing constants that minimize the forecasting error for a particular set of data values.

The rest of the paper is organized as follows. In Section II, we present some popular forecasting techniques. In section III, we discuss how to find out optimal smoothing constants by presenting a real life example. Finally, we draw a conclusion.

II. Existing Forecasting Techniques

In this section, we will discuss about different types of forecasting techniques such as Exponential Smoothing method and Holt's method.

Exponential Smoothing Method

Exponential smoothing is a technique that can be applied to time series data, either to produce smoothed data for presentation, or to make forecasts. The time series data themselves are a sequence of observations. The observed phenomenon may be an essentially random process or it may be an orderly. Whereas in the simple moving average, the past observations are weighted equally; Exponential smoothing assigns exponentially decreasing weights over time.

The simplest form of Exponential smoothing is given by the following formula:

$$F_{t+1} = \alpha A_t + (1-\alpha)F_t$$

Where,

F_{t+1} =Forecast for period $t+1$

$F_t =$ Forecast for the previous period

$\alpha =$ Smoothing constant and $0 < \alpha < 1$

$A_t =$ Actual demand or sales for the previous period

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We can interpret the new estimate of level may be seen as a weighted average of A_t, the most recent information of average level and F_t the previous estimate of that level. Small values of α imply that the revision of the old forecast, in light of the new demand is small; the new forecast is not very different from the previous one. The method requires an initial forecast F_1 which has to be either assumed or estimated.

Holt's Method

We have already introduced Exponential Smoothing technique which actually is known as Single Exponential method. But, Single Exponential method does not work well in some cases especially when the data contain trend or linear trend as we say. In that case, several methods were developed to overcome the difficulties involving errors in forecasting and usually they are referred to "Double Exponential Smoothing method".

One of the methods is named "Holt-Winters Double Exponential Smoothing" or only "Holt's method". It should be noted that Holt's method performs well where only trend but no seasonality exists.

Here, the time series exhibits a trend; in addition to the level component, the trend (slope) has to be estimated. The forecast, including trend for the upcoming period $t+1$, is given by

$$F_{t+1} = L_t + T_t$$

Here, L_t is the estimate of level made at the end of period t and is given by

$$L_t = \alpha A_t + (1-\alpha)F_t$$

T_t is the estimate of trend at the end of period t and is given by

$$T_t = \beta(L_t - L_{t-1}) + (1-\beta)T_{t-1}$$

β is also a smoothing constant between 0 and 1 and plays a role similar to that of α.

Initialization

The initial estimated base label L_0 is assumed from the last period observation and initial trend T_0 is the average monthly or weekly change.

$L_0 =$ Last period's observation

$T_0 =$ Average monthly or weekly increase

III. Real Life Example

In this section, we illustrate the process how to choose optimal smoothing constants by presenting a real life example.

Numerical Example

An established RMG factory in Dhaka produced different types of woven products such as Shirts, Pants and Trousers. Actual demand (thousands) of these products in different weeks is given in Table 1. We have to forecast the demand of these products for next week.

Table 1. Actual demand (thousands) for different weeks

Weeks	1	2	3	4	5	6	7	8
Demand	213	201	198	207	220	232	210	217
Weeks	9	10	11	12	13	14	15	
Demand	212	225	220	228	231	233	235	

Solution

We have to find out the optimal smoothing constants for both Exponential Smoothing method and Holt's method.

For Exponential Smoothing method

Let, the initial forecast $F_1 = 213$

Now, we take different values of smoothing constant (α) and for each value of α, we calculate MAD, MSE & MAPE. After that, we find out whether MAD, MSE & MAPE give minimum values. The above process is shown in Table 2.

Table 2. MAD, MSE & MAPE for different values of α

Smoothing constant(α)	Mean Absolute Deviation (MAD)	Mean Squared Error (MSE)	Mean Absolute Per. Error (MAPE)
0.1	9.680455	139.947	4.362777
0.2	8.701649	115.8347	3.933308
0.3	8.115074	103.2706	3.682017
0.4	7.83822	97.14333	3.571443
0.5	7.646403	94.20886	3.496522
0.54	7.58127	93.54398	3.471162
0.6	7.560794	92.91808	3.46761
0.62	7.556549	92.79351	3.467447
0.63	7.554824	92.74566	3.467519
0.65	7.552185	92.67795	3.467978
0.67	7.550649	92.64684	3.468869
0.68	7.550307	92.64488	3.469482
0.7	7.550497	92.66814	3.471057
0.75	7.556354	92.88832	3.477157
0.8	7.570566	93.35589	3.486674
0.9	7.628135	95.17835	3.517898

To find the optimal value of smoothing constant, minimum values of MAD, MSE & MAPE are selected and corresponding value of smoothing constant is the optimal value for this problem. After analyzing Table 2, we see that, the value of smoothing constant are 0.68, 0.68 & 0.62

for minimum MAD, MSE & MAPE respectively.

Minimum values of MAD, MSE & MAPE and corresponding value of smoothing constant is given in Table 3:

Table 3. Optimal value of smoothing constant (α)

Criteria	Minimum value	value of α
Mean Absolute Deviation (MAD)	7.5503	0.68
Mean Squared Error (MSE)	92.6449	0.68
Mean Absolute Per. Error (MAPE)	3.4674	0.62

From Table 3, we see that, two forecasting error techniques (MAD & MSE) provide the minimum value for the same value of smoothing constant (α) of 0.68. So,we choose 0.68 is the optimum value of smoothing constant for the given problem.

Using Exponential Smoothing method the forecast value for the optimal smoothing constant $\alpha = 0.68$ at the 16^{th}week is 233.9774.

For Holt's method

To solve the given problem by Holt's method, first of all we have to initialize the estimated base label and trend.

Initialization

Let the initial estimated base be L_0
Let the initial estimated trend be T_0
$L_0 =$ Last period's observation
$= 235$
$T_0 =$ Average monthly or weekly increase
$= 1.5714$

Since, In Holt's method, there has been used two smoothing constants α and β. So, first of all we fixed a particular value of α then calculated MAD, MSE & MAPE for the different values of β. After that we fixed another value of α then for different values of β, we compute MAD, MSE & MAPE. Continuing this process by fixing a particular value of α and changing the value of β, we get MAD, MSE & MAPE and find out whether MAD, MSE & MAPE give minimum value. The above process is shown by the Table 4 as follows:

Table 4. MAD, MSE & MAPE for different values of $\boldsymbol{\alpha}$ & $\boldsymbol{\beta}$

Smoothing constant (α)	Smoothing constant (β)	MAD	MSE	MAPE
0.1	0.2	14.5783	327.6925	6.8346
	0.5	16.7574	394.4027	7.7269
	0.7	17.2843	395.8708	7.9678
0.2	0.1	12.3726	249.4499	5.7937
	0.5	12.8469	276.0583	5.9609
0.3	0.1	11.5352	212.9352	5.3716
	0.4	10.5620	234.0118	4.8991
	0.6	10.7498	254.4428	4.9897
0.4	01	10.5735	190.2082	4.9106
	0.3	10.4269	207.989	4.8386
0.5	0.1	10.1548	175.4743	4.7162
	0.4	10.7176	208.725	4.9723
0.6	0.1	9.9241	164.8869	4.6097
	0.4	10.4707	198.7883	4.8556
	0.8	11.4969	245.58	5.3656
0.7	0.1	9.7055	156.7878	4.5073
	0.3	9.7596	177.8224	4.5268
0.8	0.1	9.4989	150.805	4.4094
	0.3	9.6998	170.7964	4.5032
	0.7	10.1079	204.1909	4.7288
0.9	0.1	9.3437	147.2213	4.3344
	0.2	9.3682	157.1288	4.3454
	0.7	10.1218	201.349	4.7262

Table 4 shows that MAD, MSE & MAPE all give minimum value for smoothing constants $\alpha = 0.9$ & $\beta = 0.1$

Now, this time we fixed a particular value of β then calculate MAD, MSE & MAPE for the different values of α. After that we fixed another value of β then for different values of α, we compute MAD, MSE & MAPE. Continuing this process by fixing a particular value of β and changing the values of α, we get MAD, MSE & MAPE and find out whether MAD, MSE & MAPE give minimum value. The process is shown by the Table 5

Table 5. MAD, MSE & MAPE for different values of $\boldsymbol{\beta}$ & $\boldsymbol{\alpha}$

Smoothing constant (β)	Smoothing constant (α)	MAD	MSE	MAPE
0.1	0.2	12.3726	249.4499	5.7937
	0.4	10.5725	190.2082	4.9106
	0.8	9.4989	150.805	4.4094
	0.9	9.3437	147.2213	4.3344
0.2	0.1	14.5783	327.6925	6.8346
	0.4	10.68062	199.8071	4.949276
	0.7	9.725479	167.1566	4.514025
0.3	0.2	13.74022	277.986	6.353802
	0.6	10.13564	186.5268	4.703188
	0.9	9.74451	166.8708	4.528847
0.4	0.3	10.56201	234.0118	4.89911
	0.7	10.08611	188.8139	4.68358
0.5	0.1	16.75736	394.4027	7.72697
	0.4	10.90762	231.9333	5.06183
	0.7	10.41219	199.4359	4.846066
0.6	0.2	12.23919	277.7021	5.682792
	0.4	11.54732	247.3003	5.355818
0.7	0.1	17.28431	395.8708	7.967848
	0.5	12.11636	252.9445	5.618356
0.8	0.2	11.8825	291.9447	5.509667
	0.6	11.49691	245.58	5.365551
0.9	0.1	16.88428	379.433	7.812259
	0.3	12.42949	302.6502	5.76008
	0.7	10.35647	228.8116	4.85873

Table 5 shows that MAD, MSE & MAPE also gives minimum value for smoothing constants $\beta = 0.1$ & $\alpha = 0.9$ From Table 4 & Table 5, we see that, MAD, MSE & MAPE provide the same minimum value for the same smoothing constants respectively. Minimum values of MAD, MSE & MAPE and corresponding value of smoothing constants are shown in Table 6:

Table 6. Finding optimal value of smoothing constants α & β

Criteria	Minimum value	value of α	value of β
Mean Absolute Deviation	9.3437	0.9	0.1
Mean Squared Error	147.2212	0.9	0.1
Mean Absolute Percentage Error	4.3344	0.9	0.1

From Table 6, we see that, MAD, MSE & MAPE all give the minimum value for the value of smoothing constants $\alpha = 0.9$ & $\beta = 0.1$; therefore, $\alpha = 0.9$ & $\beta = 0.1$ are the optimal value of smoothing constants for this problem.

Using Holt's method the forecast value for the optimal smoothing constants $\alpha = 0.9$ & $\beta = 0.1$ at the 16th week is 236.3227

The following Figure 1 represents the comparison between actual demand and corresponding forecast values.

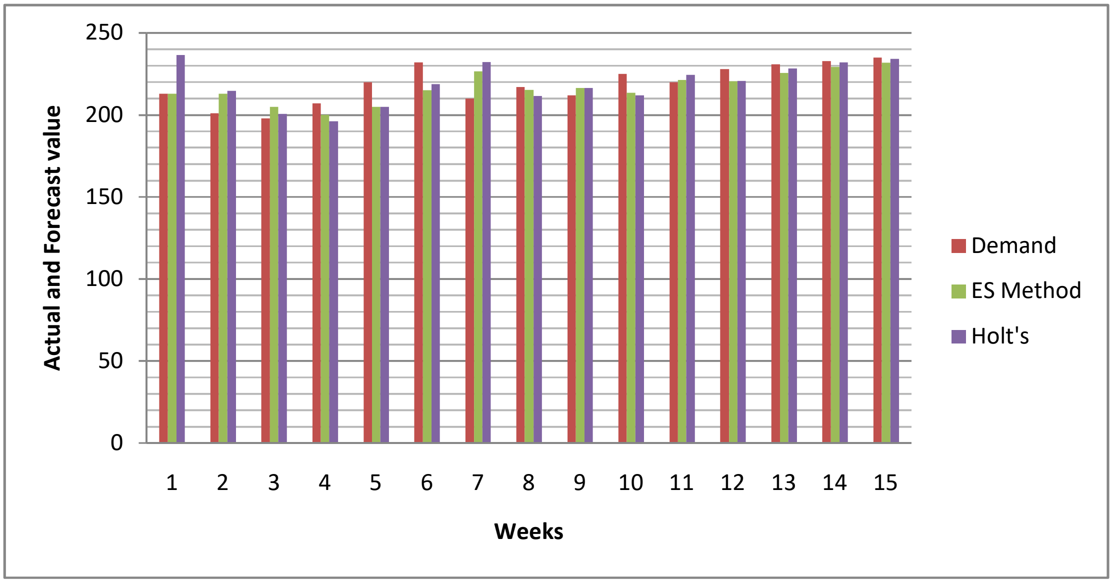

Fig.1. Comparison of actual demand and forecast value

Figure 1 represents the comparison between actual demand and forecast value for optimal smoothing constant. From Figure 1, we see that, after 12th week the forecast value move very close to actual value for both techniques and at 15th week it is almost equals to the actual value.

To solve the given problem, we use Exponential Smoothing method and Holt's method. For Exponential Smoothing method, we get the optimal smoothing constant $\alpha = 0.68$ and the corresponding forecast value for the next week is 233.9774

For Holt's method, we get the optimal smoothing constants $\alpha = 0.9$ &$\beta = 0.1$ and the corresponding forecast value for the next week is 236.3227

We have explained the procedure of choosing smoothing constants by presenting a real life example. Since our aim is to find out optimal smoothing constants for a particular set of data values, we are successfully able to find out this constant for the given real life problem. Using above procedure, we can compute the optimal value of smoothing constants for any types of data values.

IV. Conclusion

In this paper, we developed a procedure how to choose optimal smoothing constants for exponential smoothing method and Holt's method. We illustrated the solution procedure developed by us with a real life example. We therefore, hope that our procedure can help an organization to compute the optimal value of smoothing constants for particular data values that enhance the accuracy of forecasting. This work can be extended to find out the optimal smoothing constants for Holt's–Winter's Exponential method.

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